The photon element units and their relativistic properties

Brian B. K. Min[a)]
Nubron Inc., 890 Los Robles Avenue, Palo Alto, California 94306, USA

(Received 22 May 2019; accepted 14 January 2020; published online 5 February 2020)

Abstract: A set of natural units is determined from the "photon element" model of light, the outcome of an extended Compton analysis. In terms of these units, the speed of light and the electrical and Boltzmann constants are, respectively, on the order of unity, but the Planck constant is $\sim 10^{27}$ or greater and gravitational constant $\sim 10^{-59}$ or greater. This makes the photon element units less convenient than the Planck units. With the mass unit that is only $\sim 10^{-43}$ of the Planck mass, however, the photon element units can correspond better to physical realities than the Planck units. For the spacetime, a photon element forms a set of unit base vectors, a natural basis that is Lorentz covariant. There an analysis shows that (1) of the above five universal constants all are Lorentz invariants except the gravitational constant, and (2) of the five natural units (time, length, mass, electrical charge, and temperature,) only the electrical charge is a Lorentz invariant. © *2020 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-33.1.38]

Résumé: Un ensemble d'unités naturelles est déterminé à partir du modèle de "l'élément photonique "de la lumière, le résultat d'une analyse Compton étendue. En termes de ces unités, la vitesse de la lumière et les constantes électriques et de Boltzmann sont respectivement de l'ordre de l'unité, mais la constante de Planck est $\sim 10^{27}$ ou plus et la constante gravitationnelle $\sim 10^{-59}$ ou plus. Cela rend les unités d'éléments photons moins pratiques que les unités Planck. Avec l'unité de masse qui ne représente que $\sim 10^{-43}$ de la masse de Planck, cependant, les unités d'éléments photoniques peuvent mieux correspondre aux réalités physiques que les unités de Planck. Pour l'espace-temps, un élément photonique forme un ensemble de vecteurs de base unitaire, une base naturelle qui est la covariante de Lorentz. Une analyse montre que (1) des cinq constantes universelles ci-dessus toutes sont des invariants de Lorentz à l'exception de la constante gravitationnelle, et (2) des cinq unités naturelles (temps, longueur, masse, charge électrique et température), seulement la charge électrique est un invariant de Lorentz.

Key words: Space; Time; Photon; Element; Particle; Planck scale.

I. INTRODUCTION

A set of units for time, length, and mass denoted in this article as t_q, l_q, and M_q, respectively, may be uniquely derived from the universal constants, c (the speed of light), h (the Planck constant), and G (the gravitational constant) by writing their dimensional relationship[1]

$$c = l_q/t_q, \quad h = \frac{M_q l_q^2}{t_q}, \quad \text{and} \quad G = \frac{l_q^3}{M_q t_q^2}. \tag{1}$$

In addition, one can use the Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \tag{2}$$

for the electrical charge, where ε_0 is the vacuum permittivity, q_1 and q_2 are charges, r is the distance between the charges, and F is the force of interaction of the charges, along with the entropy definition

$$S = k_B \ln \Omega \tag{3}$$

for the thermodynamic temperature, where S is entropy with the unit of energy over temperature, J/K, Ω the number of states in the system, and k_B is the Boltzmann constant. Their dimensional relationships then give

$$\varepsilon_0 = \frac{1}{4\pi}\frac{q_q^2 t_q^2}{M_q l_q^3} = \frac{1}{4\pi}\frac{q_q^2}{M_q c^2 l_q}; \quad k_B = \frac{M_q l_q^2}{t_q^2 T_p} = \frac{M_q c^2}{T_p}. \tag{4}$$

The solutions to Eqs. (1) and (4) are called "Planck units" and listed in Table I. In this system of units, each unit may be normalized to unity by setting $c = h = G = k_B = 1$, and $\varepsilon_0 = 1/(4\pi)$ but they are far off the scales of elementary particles. For instance, the Planck length scale is some 20 orders of magnitude smaller than the proton radius, the mass scale some 19 orders of magnitude larger than the proton mass, and the temperature is extremely high. Although assumed to indicate the limit scales of nature, it should be remembered they are the results of a mere dimensional analysis lacking a physical model.

By an extended Compton analysis, the author showed an elemental model of the photon[2] with the mass of the "photon element" (or the "Planck element") given by

[a)]bmin@nubron.com

$$M_p = \frac{h}{c^2}\frac{1}{s} \approx 7.37 \times 10^{-51} \quad \text{kg} \approx 4.14 \times 10^{-15}\text{eV/c}^2, \tag{5}$$

which is some 43 orders of magnitude smaller than the Planck mass, M_q. In Section II, I attempt to harmonize the photon element mass, Eq. (5), with the dimensional relationship, Eqs. (1) and (4), and to find the limiting scales based upon a relativistic physical model rather than merely being dimensionally consistent.

II. PHOTON ELEMENT UNITS

Equations (1) and (5) may be harmonized in an *ad hoc* manner by first recognizing the first of Eq. (1) must hold true in any system of natural units of time, length, and mass denoted as t_p, l_p, and M_p, respectively, i.e.,

$$c = l_p/t_p. \tag{6}$$

This is necessary to ensure the speed of light is constant relativistically and will be formally justified in Section III. But from Eq. (5), we have

$$h = M_p c^2 s = n_s \frac{M_p l_p^2}{t_p}, \tag{7}$$

where we expressed the time unit, second or s, in t_p, i.e., $s = n_s t_p$, where n_s is a dimensionless integer.

Noting that G is some dimensionless constant times the dimension, we arbitrary use $k^2 n_s$ to be that constant where k is a dimensionless constant and n_s is a dimensionless integer (the role of k will become clear)

$$G = k^2 \frac{n_s l_p}{M_p} c^2 = k^2 n_s \frac{l_p^3}{M_p t_p^2}. \tag{8}$$

Equations (6)–(8) may be solved for l_p, t_p, and M_p, and the results, including the relationship with the Planck units, are also listed in Table I. For the electrical charge and thermodynamic temperature, the dimensional relationship yields [cf. Eq. (4)]

$$\varepsilon_0 = \frac{1}{4\pi}\frac{q_p^2 t_p^2}{M_p l_p^3} = \frac{1}{4\pi}\frac{q_p^2}{M_p c^2 l_p}; \quad k_B = \frac{M_p l_p^2}{t_p^2 T_p} = \frac{M_p c^2}{T_p}, \tag{9}$$

and the solutions for q_p and T_p are also listed in Table I.

The dimensionless constant k is identified to be the ratio M_p/M_q which is also equal to t_q/s

$$k = M_p/M_q = t_q/s \approx 1.35 \times 10^{-43}. \tag{10}$$

The dimensionless integer, n_s, remains to be determined hence t_p, l_p, and q_p are still unknown. The value of n_s is conjectured in Section IV, which gives the lower bound

$$n_s = c/l_p \geq\sim 3.00 \times 10^{27}.$$

That only the lower bound may be found is not the fault of the photon element model (PEM) but reflects the limited experimental data available on that scale, the very essence of it. The photon element is fundamentally related to the space-time element represented by t_p and l_p that define the speed of light in Eq. (6). In Table I, the numerical values of the photon element units including those estimated by the use of the lower bound of n_s are included and compared with those of the Planck units.

In terms of these units, the universal constants take on the numerical values $c = k_B = 1$, $h = n_s \geq\sim 3.00 \times 10^{27}$, $G = k^2 n_s \geq\sim 5.47 \times 10^{-59}$, and $\varepsilon_0 = 1/(4\pi)$. This makes the photon element units less convenient than the Planck units. With the mass unit that is only $\sim 10^{-43}$ of the Planck mass, however, the photon element units can correspond better to physical realities than the Planck units. A photon element, an outcome of the extended Compton analysis, compares with a hypothetical Planck particle, an outcome of the pre-defined Planck units.

III. LORENTZ INVARIANCE OF UNIVERSAL CONSTANTS (VIA THEIR DIMENSIONS)

The photon element units are physically based hence must be subject to the special relativistic effects such as length contraction and time dilation. Think of l_p, for instance, to be the length of a measuring rod. Physical

TABLE I. Comparisons for Planck and photon element units.

	Planck units	Photon element units
Time	$t_q = \sqrt{\frac{hG}{c^5}} \approx 1.35 \times 10^{-43}$ s	$t_p = \frac{1}{kn_s}\sqrt{\frac{hG}{c^5}} \leq\sim 3.34 \times 10^{-28}$ s
Length	$l_q = \sqrt{\frac{hG}{c^3}} \approx 4.05 \times 10^{-35}$ m	$l_p = \frac{1}{kn_s}\sqrt{\frac{hG}{c^3}} \leq\sim 1.00 \times 10^{-19}$ m
Mass	$M_q = \sqrt{\frac{hc}{G}} \approx 5.46 \times 10^{-8}$ kg	$M_p = k\sqrt{\frac{hc}{G}} \approx 7.37 \times 10^{-51}$ kg
Charge	$q_q = \sqrt{4\pi\varepsilon_0 hc} \approx 4.70 \times 10^{-18}$ C	$q_p = \sqrt{4\pi\varepsilon_0 hc/n_s} \leq\sim 8.59 \times 10^{-32}$ C
Temperature	$T_q = \sqrt{\frac{hc^5}{Gk_B^2}} \approx 3.55 \times 10^{32}$ K	$T_p = k\sqrt{\frac{hc^5}{Gk_B^2}} \approx 4.80 \times 10^{-11}$ K

constants have dimensions comprising time, length, mass, etc., which are individually subject to the special relativistic effects.

The universal constants used in Section II may be referred to as the "rest" universal constants. A universal constant (via its dimension) that remains invariant under Lorentz transformations may be called a Lorentz constant. It is distinguished from the Lorentz scalar which is not necessarily a physical constant. The only Lorentz constant that has been experimentally well verified is the speed of light, c, Eq. (6). It is then interesting to examine the Lorentz (in)variance of the other universal constants, h, G, ε_0, and k_B in Eqs. (1) and (9).

The Lorentz transformation of the coordinates x^μ in an inertial frame, say K, to x'^μ in K' moving with the velocity v in x direction relative to K, may be expressed as following:

$$x'^\mu = \Lambda^\mu_\nu x^\nu, \tag{11}$$

where $x^\mu \equiv (x^0, x^1, x^2, x^3) \equiv (ct, x, y, z)$ (notations interchangeable) and

$$\Lambda^\mu_\nu = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad \mu, \nu = 0, 1, 2, 3, \tag{12}$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. An abstract vector A, or $A^\kappa (\kappa = 0, 1, 2, 3)$ to write its four-components explicitly, can be written as linear combination of the unit base vectors (I adopt Refs. 3–5 for the notations with substantial deviations)

$$\begin{aligned} A^\kappa &\equiv V^\mu \hat{e}^\kappa_{(\mu)} = V^0 \hat{e}^\kappa_{(0)} + V^1 \hat{e}^\kappa_{(1)} + V^2 \hat{e}^\kappa_{(2)} + V^3 \hat{e}^\kappa_{(3)}, \\ &= V^0 \hat{e}^{(0)\kappa} - V^1 \hat{e}^{(1)\kappa} - V^2 \hat{e}^{(2)\kappa} - V^3 \hat{e}^{(3)\kappa}, \end{aligned} \tag{13}$$

where V^μ and $\hat{e}^\kappa_{(\mu)}$ are some conveniently chosen components and unit base vectors, respectively. The parenthesis around an index indicates that this is an individual base vector; its components are indicated by an additional index, as in $\hat{e}^\kappa_{(\mu)}$.

Now Eq. (6), $c = l_p/t_p$, suggests the spacetime be elementary (or discrete) with t_p and l_p to be the units from the fundamental elements. An elemental vector may be defined as a combination of the integer components and the base vectors. We will then redefine the abstract vector A, Eq. (13), to be a linear combination of integer variables, n^μ, and the base vectors, $\hat{l}_{(\mu)}$,

$$\begin{aligned} A^\kappa &\equiv n^\mu \hat{l}^\kappa_{(\mu)} = n^0 \hat{l}^\kappa_{(0)} + n^1 \hat{l}^\kappa_{(1)} + n^2 \hat{l}^\kappa_{(2)} + n^3 \hat{l}^\kappa_{(3)}, \\ &= n^0 \hat{l}^{(0)\kappa} - n^1 \hat{l}^{(1)\kappa} - n^2 \hat{l}^{(2)\kappa} - n^3 \hat{l}^{(3)\kappa}. \end{aligned} \tag{14}$$

The Lorentz transformation of the four-vector A, Eq. (14), may be expressed as

$$A'^\kappa \equiv n'^\mu \hat{l}'^\kappa_{(\mu)}, \tag{15}$$

which may go by applying the transformation to the vector components or to the base vectors as following:

$$\begin{aligned} A'^\kappa &= (\Lambda^\mu_\nu n^\nu) \hat{l}^\kappa_{(\mu)} = n^\nu (\Lambda^\mu_\nu \hat{l}^\kappa_{(\mu)}), \\ &= (\gamma n^0 - \beta\gamma n^1) \hat{l}^\kappa_{(0)} + (-\beta\gamma n^0 + \gamma n^1) \hat{l}^\kappa_{(1)} \\ &\quad + n^2 \hat{l}^\kappa_{(2)} + n^3 \hat{l}^\kappa_{(3)}. \end{aligned} \tag{16}$$

Note that this compares with Schild[6] or Jensen and Pommerenke[7] who investigated only the integral transformation (versus base vector transformation.)

The next step to establish the elemental spacetime (at least in the mathematical sense) is to consider the photon element to form the basis

$$\begin{aligned} &\hat{l}^\kappa_{(0)} = \begin{pmatrix} ct_p \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad \hat{l}^\kappa_{(1)} = \begin{pmatrix} 0 \\ l_p \\ 0 \\ 0 \end{pmatrix}; \\ &\hat{l}^\kappa_{(2)} = \begin{pmatrix} 0 \\ 0 \\ l_p \\ 0 \end{pmatrix}; \quad \hat{l}^\kappa_{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ l_p \end{pmatrix}. \end{aligned} \tag{17}$$

Equation (16) then becomes

$$\begin{aligned} A'^\kappa &= (\gamma n^0 - \beta\gamma n^1) \begin{pmatrix} ct_p \\ 0 \\ 0 \\ 0 \end{pmatrix} + (-\beta\gamma n^0 + \gamma n^1) \begin{pmatrix} 0 \\ l_p \\ 0 \\ 0 \end{pmatrix} \\ &\quad + n^2 \begin{pmatrix} 0 \\ 0 \\ l_p \\ 0 \end{pmatrix} + n^3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ l_p \end{pmatrix}, \\ &= \begin{pmatrix} (\gamma n^0 - \beta\gamma n^1) ct_p \\ (-\beta\gamma n^0 + \gamma n^1) l_p \\ n^2 l_p \\ n^3 l_p \end{pmatrix}. \end{aligned} \tag{18}$$

The Lorentz transformation may also be performed to the base vectors individually

$$A'^{\kappa} = n'^{\nu}\hat{l}'^{\kappa}_{(\nu)} = n^{\nu}(\Lambda^{\kappa}_{\mu}\hat{l}^{\mu}_{(\nu)}) \text{ where } n'^{\nu} = n^{\nu} \text{ and } \hat{l}'^{\kappa}_{(\nu)} = \Lambda^{\kappa}_{\mu}\hat{l}^{\mu}_{(\nu)}, \tag{19}$$

and this must be consistent with Eq. (18).

We compute

$$\hat{l}'^{\kappa}_{(0)} \equiv \begin{pmatrix} ct'_p \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} ct_p \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma ct_p \\ -\beta\gamma ct_p \\ 0 \\ 0 \end{pmatrix}, \tag{20}$$

and similarly

$$\hat{l}'^{\kappa}_{(1)} = \begin{pmatrix} 0 \\ l'_p \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\beta\gamma l_p \\ \gamma l_p \\ 0 \\ 0 \end{pmatrix}; \quad \hat{l}'^{\kappa}_{(2)} = \begin{pmatrix} 0 \\ 0 \\ l'_p \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ l_p \\ 0 \end{pmatrix}; \quad \hat{l}'^{\kappa}_{(3)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ l'_p \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ l_p \end{pmatrix}. \tag{21}$$

We must be able to express the vector A and its transformation A' by using the base vectors, Eq. (17), and their Lorentz transformation, Eqs. (20) and (21), respectively, i.e. from Eq. (14),

$$A^{\kappa} = n^{\mu}\hat{l}^{\kappa}_{(\mu)} = n^0 \begin{pmatrix} ct_p \\ 0 \\ 0 \\ 0 \end{pmatrix} + n^1 \begin{pmatrix} 0 \\ l_p \\ 0 \\ 0 \end{pmatrix} + n^2 \begin{pmatrix} 0 \\ 0 \\ l_p \\ 0 \end{pmatrix} + n^3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ l_p \end{pmatrix} = \begin{pmatrix} n^0 ct_p \\ n^1 l_p \\ n^2 l_p \\ n^3 l_p \end{pmatrix}$$

and from Eq. (19)

$$A'^{\kappa} = \begin{pmatrix} n'^0 ct'_p \\ n'^1 l'_p \\ n'^2 l'_p \\ n'^3 l'_p \end{pmatrix} = \begin{pmatrix} n^0 ct'_p \\ n^1 l'_p \\ n^2 l'_p \\ n^3 l'_p \end{pmatrix} = n^0 \begin{pmatrix} ct'_p \\ 0 \\ 0 \\ 0 \end{pmatrix} + n^1 \begin{pmatrix} 0 \\ l'_p \\ 0 \\ 0 \end{pmatrix} + n^2 \begin{pmatrix} 0 \\ 0 \\ l'_p \\ 0 \end{pmatrix} + n^3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ l'_p \end{pmatrix}$$
$$= n^0 \begin{pmatrix} \gamma ct_p \\ -\beta\gamma ct_p \\ 0 \\ 0 \end{pmatrix} + n^1 \begin{pmatrix} -\beta\gamma l_p \\ \gamma l_p \\ 0 \\ 0 \end{pmatrix} + n^2 \begin{pmatrix} 0 \\ 0 \\ l_p \\ 0 \end{pmatrix} + n^3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ l_p \end{pmatrix} = \begin{pmatrix} \gamma n^0 ct_p - \beta\gamma n^1 l_p \\ -\beta\gamma n^0 ct_p + \gamma n^1 l_p \\ n^2 l_p \\ n^3 l_p \end{pmatrix} \tag{22}$$

By comparing Eq. (18) with (22), we then must have

$$\begin{pmatrix} (\gamma n^0 - \beta\gamma n^1) ct_p \\ (-\beta\gamma n^0 + \gamma n^1) l_p \\ n^2 l_p \\ n^3 l_p \end{pmatrix} = \begin{pmatrix} \gamma n^0 ct_p - \beta\gamma n^1 l_p \\ -\beta\gamma n^0 ct_p + \gamma n^1 l_p \\ n^2 l_p \\ n^3 l_p \end{pmatrix} \tag{23}$$

and find Eq. (23) will hold true only if

$$ct_p = l_p. \tag{24}$$

Equation (6) which was merely a result of the dimensional analysis has now been verified by the elemental model of the spacetime and its Lorentz transformation. We find Eq. (6) to be the fundamental property that defines the spacetime at least in this mathematical sense.

From Eq. (22), the invariant of the four vector is (here, for convenience, we use additional notations $(n^0, n^1, n^2, n^3) \equiv (n_t, n_x, n_y, n_z)$ with a metric +- - -)

$$
\begin{aligned}
&(n_t'^2 - n_x'^2 - n_y'^2 - n_z'^2)l_p'^2 \\
&\quad = (n_t\gamma c t_p - n_x\beta\gamma l_p)^2 - (-n_t\beta\gamma c t_p + n_x\gamma l_p)^2 \\
&\qquad - (n_y l_p)^2 - (n_z l_p)^2, \\
&\quad = (n_t^2 - n_x^2 - n_y^2 - n_z^2)l_p^2,
\end{aligned} \tag{25}
$$

which tells us that $n_t^2 - n_r^2$ (where $n_r^2 \equiv n_x^2 + n_y^2 + n_z^2$) is an invariant and an observer will be oblivious to the transformation, $l_p \to l_p'$, a consequence of Eq. (24). The speed of light is constant regardless of the inertial frame of reference, because the spacetime is elemental satisfying Eq. (24), a profound finding. Finally, from Eqs. (18) and (22), we note the transformation

$$
\begin{pmatrix} n^0 c t_p \\ n^1 l_p \\ n^2 l_p \\ n^3 l_p \end{pmatrix} \to \begin{pmatrix} (\gamma n^0 - \beta\gamma n^1) c t_p \\ (-\beta\gamma n^0 + \gamma n^1) l_p \\ n^2 l_p \\ n^3 l_p \end{pmatrix}
= \begin{pmatrix} n^0\left(1 - \dfrac{n^1}{n^0}\beta\right)(\gamma c t_p) \\ n^1\left(1 - \dfrac{n^0}{n^1}\beta\right)(\gamma l_p) \\ n^2 l_p \\ n^3 l_p \end{pmatrix} \tag{26}
$$

and see that the Lorentz transformation applies the boost factor, γ, to both t_p and l_p. We may, therefore, use $t_p \to \gamma t_p$ and $l_p \to \gamma l_p$ to mathematically determine the Lorentz invariance of any physical quantities. For example, from Eq. (6),

$$
c = l_p/t_p \to \gamma l_p/(\gamma t_p) = l_p/t_p. \tag{27}
$$

According to relativistic quantum theories, h is Lorentz-constant and from Eq. (7), it holds only if M_p transform to M_p/γ when in motion. The latter possibility was discussed in Refs. 8 and 9, where we used the energy momentum relation in $1/\gamma$-metric as well as in γ-metric. It should be noted that the usual relativistic energy momentum relation is derived from the Minkowski four-velocity which must be divided by γ to obtain the observable velocity. With no such consideration, we have been used to think a (rest) mass is an invariant quantity but we are now forced to acknowledge that the Lorentz transformation has the rest mass M_p transform to M_p/γ when in motion. In fact, this transformed mass is consistent with the proper mass $\gamma M_p = E_p/c^2$ appearing in the usual Minkowski space energy-momentum relation. On this basis, we get

$$
h = M_p c^2 s = M_p c^2 n_s t_p \to (M_p/\gamma)c^2 n_s \gamma t_p = h, \tag{28}
$$

and

$$
G = k^2 \frac{n_s l_p}{M_p} c^2 \to k^2 \frac{n_s \gamma l_p}{M_p/\gamma} c^2 = \gamma^2 G, \tag{29}
$$

Our Lorentz boost rule now becomes

$$
c \to c; \quad h \to h; \quad G \to \gamma^2 G. \tag{30}
$$

Both the Planck units and the photon element units listed in Table I preserve not only the dimensional relationship but also the Lorentz variance for, if we replace G with $\gamma^2 G$, then t_p, l_p, and M_p transform to γt_p, γl_p, and M_p/γ, respectively, consistent with the above argument. Most remarkably, we discover that the gravitational constant, G, is not a Lorentz invariant universal constant. Possible physical justifications are discussed in Section V C. The implication of this result, however, warrants a further exploration beyond the scope of this paper.

Likewise, the electrical constant (vacuum permittivity) transforms

$$
\varepsilon_0 = \frac{n_s}{4\pi} \frac{q_p^2}{hc}; \quad \varepsilon_0 \to \varepsilon_0 \tag{31}
$$

to confirm it is Lorentz constant as has been established by the electrodynamics. The elemental thermal energy $k_B T_p = M_p c^2 = k\sqrt{\frac{hc^5}{G}}$ will transform $k_B T_p \to k_B T_p/\gamma$, hence

$$
k_B = \frac{M_p c^2}{T_p} = k\sqrt{\frac{hc^5}{G T_p^2}}; \quad k_B \to k_B. \tag{32}
$$

The Boltzmann's constant k_B is therefore Lorentz constant while the photon element temperature transforms as $T_p \to T_p/\gamma$. The interpretation of this temperature transformation is that the proper elemental temperature will rise as $T_p \to \gamma T_p$.

In Table II, the photon element units expressed in universal constants are listed along with their Lorentz (in) variant properties.

The above results deserve a careful interpretation. The Lorentz transformation of the rest time t_p is γt_p (for instance, for a moving particle seen from the laboratory frame) which gives the proper (or invariant) time $\tau_p = t_p/\gamma$ (for a moving particle in its own frame). Equation (24) then ensures the

TABLE II. Photon element units (expressed in universal constants) and their Lorentz (in)variance.

Rest photon element units	Expressed in universal constants	Lorentz transformed	Proper (invariant) quantities
t_p	$\frac{1}{kn_s}\sqrt{\frac{hG}{c^5}}$	γt_p	t_p/γ
l_p	$\frac{1}{kn_s}\sqrt{\frac{hG}{c^3}}$	γl_p	l_p/γ
M_p	$k\sqrt{\frac{hc}{G}}$	M_p/γ	γM_p
q_p	$\sqrt{4\pi\varepsilon_0 hc/n_s}$	q_p	q_p
T_p	$k\sqrt{\frac{hc^5}{Gk_B^2}}$	T_p/γ	γT_p

Lorentz transformation of the rest length l_p to be γl_p which gives the proper length l_p/γ; The Lorentz transformation of the rest mass M_p is M_p/γ which gives the proper mass γM_p, etc. The transformed kinetic energy is $(1-1/\gamma)M_pc^2$ versus the Minkowski space, invariant kinetic energy $(\gamma-1)M_pc^2$; note, however, that in both cases, the low speed limit produces the Newtonian kinetic energy $M_pv^2/2$.

IV. ESTIMATES OF THE ELEMENTAL LENGTH AND n_s

Under the Lorentz transformation, the elemental length l_p may contract to an observable value, i.e., $l_p \to l_p/\gamma$, which makes the lower limit of the length scale to be immaterial. We therefore only consider the upper limit of l_p in the elemental spacetime.

In the elemental spacetime, we note that the elemental length, l_p, must be the theoretical lower limit for the wavelength of electromagnetic waves. Researchers project the lower limit value, if any, of the ultrahigh energy gamma rays[10–13] in the range

$$\lambda_{\gamma-ray} \sim 1.00 \times 10^{-19}\,\mathrm{m} - 1.00 \times 10^{-25}\,\mathrm{m}.$$

We, therefore, put the upper bound of l_p (or the upper bound of t_p)

$$l_p \leq\sim 1.00 \times 10^{-19}\,\mathrm{m}\,\left(\text{or } t_p = l_p/c \leq\sim 3.34 \times 10^{-28}\,\mathrm{s}\right),$$

which gives the lower bound n_s

$$n_s = c/l_p \geq\sim 3.00 \times 10^{27}.$$

For a comparison, the diameter of neutrinos[14] is estimated to be

$$\Lambda_{\text{D}-\text{neutrino}} \sim 1.00 \times 10^{-19}\,\mathrm{m}.$$

High energy cosmic rays[15] are known to be proton particles with the energy greater than even that of the highest gamma rays. They include: Greisen–Zatsepin–Kuzmin (GZK) limit with $\lambda_{gzk} \sim 2.48 \times 10^{-26}$ m; Ultra High Energy Cosmic Rays with $\lambda_{uhe} \sim 4.13 \times 10^{-27}$ m; and 300 EeV cosmic rays with $\lambda_{300EeV} \sim 1.24 \times 10^{-27}$ m. These are the De Broglie wavelengths of the proton matter waves and their energy levels do not represent the electromagnetic waves carried by the elemental spacetime, since a proton is a massive, composite particle travelling at less than the speed of light.

V. DISCUSSION

A. Experimental verification

The photon element mass is definitely determined from the Planck constant. In that sense, an experiment has already been available for over a century,[16] yet we have not recognized that the existence of the Planck constant, the quantum of the electromagnetic action, may mean the existence of the photon element in the sense described here. What about the size of the photon element? Can we perform an experiment to determine the only unknown of the PEM, n_s, that defines the granularity of the time and length? Theoretically this limits the wavelength of the electromagnetic waves to $\sim l_p$ defining an upper bound of their energy. Therefore, if an experiment can establish that there is an upper bound for the gamma ray energy, the time and length would have been found granular. This is an experiment to help verify the PEM.

Until recently, no gamma ray energy greater than about 100 TeV[17] has been observed, but since the submission of this manuscript for publication, the Tibet Air Shower Gamma (ASγ) Collaboration reports the findings of 24 photon-initiated showers with photon energies above 100 TeV—one of which registering 450 TeV.[18] This pushes the upper bound of the length unit from $l_p \sim 1 \times 10^{-19}$ m estimated in Section IV down to about $l_p \sim 1 \times 10^{-21}$ m or from $n_s \geq\sim 3.00 \times 10^{27}$ to about $n_s \geq\sim 3.00 \times 10^{29}$.

Experiments should then focus on extending the observed electromagnetic spectrum on the upper energy end, preferably in a laboratory. The energy of Large Hadron Collider (LHC) built by the European Organization for Nuclear Research (CERN) is limited to $\sim$10 TeV and may not able to reach the necessary energy level, >450 TeV, even if $\gamma-$rays can be generated. Continued observations from the cosmic sources such as the Tibet ASγ Collaboration may be more promising.

Finally, the presence of the photon elements at the fundamental scale may mean our vacuum (or the quantum field) is filled with a kind of granular materials[19] causing quantum activities. We can then imagine a space where even these materials are absent, a true vacuum where no quantum activities may occur. Can we create and observe a true vacuum in a laboratory? Such an experiment, however, may be extremely difficult to perform, if not impossible.

B. Lorentz invariance on the photon element scale

This discussion is speculative. We note that the spacetime granularity in the PEM is subtly but fundamentally different from the absolute spacetime granularity assumed by some theories of quantum gravity (QGs). This is because unlike these QGs where the spacetime itself is granular, the photon elements propagate as waves on a conjectured granular material that fills the space and may itself flow. This material is thought to give the energy (or mass) of the cosmological constant, an essential ingredient of the modern cosmology first postulated by Einstein. This conjectured granular material that the author calls the gamma element is discussed in Ref. 19, where it is modeled as perfect fluid when filling the space.

With the constraint $l_p/t_p = c$ that must hold by definition, the speed of light remains constant to an observer even on the photon element scale. Unlike the QGs, the photon elementary distance is not absolute but subject to the special relativity, $l_p \to \gamma l_p$. There is no privileged reference frame. The Michelson–Morley experiment will still hold and the principle of relativity will not be disrupted.

The QG theories, such as the string theory and loop QG, assume an absolute granularity of spacetime on the Planck scale $\sim 4 \times 10^{-35}$ m where a Lorentz invariance violation may occur. By studying the polarization of gamma ray photons from the supernova GRB041219A, Laurent *et al.* pushed the constraints on Lorentz invariance violation to about $\sim 1 \times 10^{-45}$ m.[20] Rather than pushing the granularity to this small scale, this result probably indicates that there is no Lorentz invariance violation after all, favoring the PEM.

Our tool for observation is light. Below the photon element scale, therefore, we do not have a tool to observe with. It does not mean that there is no physics there but we have no means to directly access.

C. Gravitational constant

How do we physically justify the universal gravitation constant,G, Lorentz-transforming as $G \to \gamma^2 G$? I will briefly describe some conjectured consequences based upon work in progress: A full account would be far beyond the scope of this paper.

Consider the electron and proton moving relative to each other at a significant fraction of light speed in a hydrogen atom or the quarks inside a proton at even higher relative velocities. The gravitational forces may no longer be negligible: $G \to \gamma^2 G$ (unlike G alone) is thought to be able to produce the kind of relativistic gravitational forces that are consistent with the electromagnetic and strong forces, respectively. This would be a useful step for the unification of gravitational and those other forces, and warrants further investigation with detailed calculations.

On a larger scale, the transformation of the Newtonian gravitational potential energy at a distance r from a body of mass M given by

$$\Phi(r) = -\frac{GM}{r}\gamma \quad \text{where} \quad \gamma = \left[1 - (\dot{r}/c)^2\right]^{-1/2} \tag{33}$$

(this is seen by the length contraction of r or may be argued by more detailed discussion of Lorentz transformation of this paper) was applied to account for the special relativistic effect for the expanding universe.[19] The result of Eq. (33) is shown to closely agree with that derived by applying the theory of general relativity.

VI. SUMMARY AND CONCLUSION

A new natural system of units has been obtained in an *ad hoc* manner from the photon element that is an outcome of a relativistic extension of Compton analysis. It relates with the Planck units via a couple of dimensionless constants.

In the system of photon element units, one may set $c = k_B = 1,\ h = n_s,\ G = k^2 n_s$ and $\varepsilon_0 = 1/(4\pi)$ to define the natural units (i.e., each of all five units is unity.) In this system of units, the mass and temperature scales are definitely known, $\approx 7.37 \times 10^{-51}$ kg and $\approx 4.80 \times 10^{-11}$ K, respectively, but the time, length, and electrical charge scales are dependent on the constant n_s and can only be bounded at this time: $\leq\sim 3.34 \times 10^{-28}$ s, $\leq\sim 1.00 \times 10^{-29}$ m, and $\leq\sim 8.59 \times 10^{-32}$ C, respectively. These upper bounds may be tightened by future experiments. Note that the mass scale is about 19 orders of magnitude smaller than the proton mass and the length scale upper bound about four orders of magnitude smaller than the proton radius.

More formally, it has been shown the elemental spacetime naturally gives these units at the "photon element scale." A consideration of Lorentz (in) variance shows that the fundamental physical constants must transform

$$c \to c;\quad h \to h;\quad G \to \gamma^2 G;\quad \varepsilon_0 \to \varepsilon_0;\quad \text{and}\quad k_B \to k_B.$$

It has been shown the speed of light, c, and the Planck constant, h, the universal constants for the special relativity and the quantum mechanics, respectively, are Lorentz invariant. The electrical and Boltzmann constants, ε_0 and k_B, respectively, are also Lorentz-invariant, but the gravitational constant is not, which gives the Lorentz transformation of the photon element units

$$t_p \to \gamma t_p,\quad l_p \to \gamma l_p,\quad M_p \to M_p/\gamma,\quad q_p \to q_p,\quad T_p \to T_p/\gamma$$

and their Lorentz invariant proper quantities

$$t_p \to t_p/\gamma,\quad l_p \to l_p/\gamma,\quad M_p \to \gamma M_p,\quad q_p \to q_p,\quad T_p \to \gamma T_p.$$

Finally, other systems of natural units that are in use should be mentioned for comparisons. Like the Planck particle being the hypothetical physical model for the Planck units, the electron and proton are the physical models for the atomic and quantum chromodynamics (or strong) units, respectively, as they correspond well to the scales of atoms and nuclei, respectively. The photon element is the physical model for the photon element units, undoubtedly one of the most fundamental physical units discovered to this date if the present theory is correct. The real significance of the system of photon element units, however, is that as nature's most fundamental physical units of measurement it is on a vastly different scale compared to the system of Planck units even though both originate from the common ground of Planck constant.

ACKNOWLEDGMENTS

The author thanks an anonymous reviewer for the valuable comments. Section V addresses some of the issues raised.

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