Relativistic fields allowing spontaneous transformation between mass and charge

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Abstract: A quantum wave equation that extends the Schrödinger equation with its probability density into the relativistic regime is explored to obtain the "modified" fields of Klein–Gordon, Dirac, Proca, and Higgs, respectively. Unlike the original, the modified Dirac field is shown to include a massless state in addition to the particle and antiparticle states. A gauge transformation for the modified Klein–Gordon field (MKG) gives scalar bosons that allow transformation between a massive state and a massless, charged state. It also gives massive vector bosons owing to the spontaneously broken space–time symmetry, a feature of the MKG, rather than the Brout–Englert–Higgs mechanism.
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Résumé: Une équation d'onde quantique qui étend l'équation de Schrödinger avec sa densité de probabilité dans le régime relativiste est explorée pour obtenir les champs «modifiés» de Klein-Gordon, Dirac, Proca et Higgs, respectivement. Contrairement à l'original, le champ de Dirac modifié est montré pour inclure un état sans masse en plus des états de particule et anti-particule. Une transformation de jauge pour le champ modifié de Klein-Gordon (MKG) donne des bosons scalaires qui permettent la transformation entre un état massif et un état chargé sans masse. Il donne aussi des bosons vecteurs massifs en raison de la symétrie spatio-temporelle brisée spontanément, une caractéristique du MKG, plutôt que du mécanisme de Brout-Englert-Higgs (BEH).

Key words: Schrödinger; Relativistic; Quantum Field; Klein–Gordon; Dirac; Proca; Higgs.

I. INTRODUCTION

The Lagrangian formalism in the quantum field theories describes the massive scalar boson field by the Lagrangian of the Klein–Gordon (KG) equation, the spin half fermion field by the Lagrangian of the Dirac equation, and the massive vector boson field by the Lagrangian of the Proca equation, etc. The relativistic quantum wave equations in these theories use the quantum prescriptions applied to the total energy, E, which is the sum of the relativistic external kinetic energy and the internal (rest) energy, and to the relativistic momentum, P. In general, however, the external kinetic energy and the internal energy (or mass) originate differently and may be difficult to describe by a single set of wave equations. In this paper, I show a novel, kinetic energy-operated quantum wave equation resolves this fundamental problem and provides corrections to the known quantum wave equations, hence to their Lagrangians. I then show some consequences of these corrections to the fields of KG, Dirac, Proca, and Higgs, respectively, by the use of the gauge transformation formalism.

Gauge theories account for the dynamics of most elementary particles in the Standard Model. The principle of local gauge invariance leads to the fundamental quantum field theories, encompassing the Feynman rules, quantum electrodynamics, and quantum chromodynamics. It predicts the existence of the Higgs boson and the mass of the W and Z bozons, the mediator of weak forces, by spontaneous symmetry breaking via the Higgs mechanism. The existence of the Higgs boson was confirmed by experiment.

With the Higgs mechanism, "mass" is created from the potential energy transferred from the Higgs field to fundamental particles. In the present work, transformation between the mass-equivalent energy (Mc^2) and electrical potential energy (Vq) occurs, which the author refers to loosely as the transformation between mass and charge. The relativistic energy-momentum relation is preserved throughout the spontaneous transformation between mass and charge.

II. RELATIVISTIC ENERGY-MOMENTUM RELATION

In this and Section III, I extract some prerequisites from the author's previous work.[1] I can write the relativistic energy–momentum relation in terms of the total energy, E, and momentum, P, of a particle[2–8]

$$E^2 = P^2c^2 + M^2c^4, \tag{1}$$

where c is the speed of light and M is the mass of the particle.

Now the relativistic kinetic energy, T, may be written as

$$T = E - Mc^2. \tag{2}$$

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We can then rewrite the energy-momentum relation, Eq. (1), in terms of the kinetic energy and momentum,

$$T^2 + 2Mc^2T = P^2c^2. \tag{3}$$

If I define $\mathscr{E} \equiv Mc^2$, the internal energy (many authors call this the rest energy) and $\mathscr{P} \equiv M\upsilon$ to be the nonrelativistic momentum, I can then call $E \equiv \gamma Mc^2 = \gamma\mathscr{E}$ to be the relativistic total energy and $P = \gamma M\upsilon = \gamma\mathscr{P}$ to be the relativistic momentum where $\gamma = 1\Big/\sqrt{1-\frac{\upsilon^2}{c^2}}$ is the Lorentz factor and v the velocity of the particle.

The energy-momentum relation, Eq. (1), may then be rewritten

$$\mathscr{E}^2 = \mathscr{P}^2c^2 + \left(\frac{M}{\gamma}\right)^2 c^4, \tag{4}$$

in terms of the internal energy, nonrelativistic momentum, and mass. I note that Eqs. (1) and (4) are of the same form except the mass M is replaced with M/γ, a relativistic mass or the mass normalized by the Lorentz factor, with $0 \le 1/\gamma \le 1$. We can say Eq. (1) is written in the γ-metric whereas Eq. (4) is in the $1/\gamma$ metric. As the speed of the particle approaches that of light, Eq. (1) may blow up but Eq. (4) behaves well thanks to the $1/\gamma$ metric which instead makes the relativistic mass vanish. It is crucial to have this metric to characterize a particle. For example, each of the electron's orbits in an atom has a particular angular velocity and radius hence a characteristic γ and $1/\gamma$ values.

In the same way, Eq. (3) may be rewritten as

$$\mathscr{T}^2 + \frac{2Mc^2}{\gamma}\mathscr{T} = \mathscr{P}^2c^2, \tag{5}$$

where $\mathscr{T} \equiv T/\gamma$. If a quantum wave equation is built based upon Eq. (4) or Eq. (5), the γ-metric counterpart for a scripted quantity, for instance, T for $\mathscr{T}$ or E for $\mathscr{E}$, may easily be recovered by multiplying γ.

III. MODIFIED KLEIN–GORDON EQUATION

The quantum prescriptions are based upon the de Broglie's theory[6] commonly expressed by $\boldsymbol{P} = \hbar\boldsymbol{k}$ and $E = \hbar\omega$ (or $\mathscr{P} = \hbar\mathscr{k}$ and $\mathscr{E} = \hbar\omega$ in the $1/\gamma$ metric) where $\hbar$ is the reduced Planck constant, $\boldsymbol{k}$ (or $\mathscr{k}$) is the wave number and ω (or ω) is the angular frequency.[9] The bold face indicates a three-vector. To isolate the external motion from the internal motion of a particle, however, I replace the above total energy with the relativistic kinetic energy, hence I have $\boldsymbol{P} = \hbar\boldsymbol{k}$ and $T = \hbar\omega$ (or $\mathscr{P} = \hbar\mathscr{k}$ and $\mathscr{T} = \hbar\omega$ in the $1/\gamma$ metric). Substituting $\mathscr{T}$ by $i\hbar(\partial/\partial t)$ and $\mathscr{P}$ by $i\hbar\nabla$ in Eq. (5) and operating on a scalar function Φ, I then obtain

$$i\hbar\frac{\partial}{\partial t}\Phi = \frac{\hbar^2\gamma}{2M}\Box\Phi, \tag{6}$$

where $\Box \equiv (1/c^2)(\partial^2/\partial t^2) - \nabla^2$, the d'Alembertian, or in the tensor notation

$$\partial_\mu\partial^\mu\Phi = i\alpha\partial_0\Phi, \tag{7}$$

where

$$\alpha \equiv \frac{2Mc}{\hbar\gamma}. \tag{8}$$

This is the kinetic energy-operated, relativistic quantum wave equation in the $1/\gamma$ metric, an extension of the Schrödinger equation in the free field. Note that if I replace the d'Alembertian with $-\nabla^2$ and take the nonrelativistic limit of the relativistic mass $M/\gamma \to M$ I recover the Schrödinger equation. Conversely, the relativistic extension of the Schrödinger equation in the $1/\gamma$ metric may be simply constructed by replacing $-\nabla^2$ in the Schrödinger equation with $\Box \equiv \partial_\mu\partial^\mu$ and the mass M with the relativistic mass M/γ.

We now define a unit four vector

$$I^\mu = I_0^\mu + I_1^\mu + I_2^\mu + I_3^\mu, \tag{9}$$

where

$$I^\mu \equiv \begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \quad I_0^\mu \equiv \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad I_1^\mu \equiv \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad I_2^\mu \equiv \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \quad I_3^\mu \equiv \begin{pmatrix}0\\0\\0\\1\end{pmatrix}. \tag{10}$$

When applied to the four derivative, it is understood that

$$\begin{aligned} &I^\mu\partial_\mu = \partial_0 - \partial_1 - \partial_2 - \partial_3 \\ &I_0^\mu\partial_\mu = \partial_0; \quad I_1^\mu\partial_\mu = -\partial_1; \text{ etc.} \end{aligned} \tag{11}$$

This allows Eq. (7) to be rewritten in a more maneuverable form

$$\partial_\mu\partial^\mu\Phi = i\alpha I_0^\mu\partial_\mu\Phi. \tag{12}$$

The above is the modified Klein–Gordon (MKG) equation, a new relativistic quantum wave equation for spin zero massive particles that reduces to the Schrödinger equation in the nonrelativistic limit, $\Box \sim -\nabla^2$ and $\gamma \sim 1$. In the new approach, it replaces the KG equation

$$\partial_\mu\partial^\mu\Phi + \left(\frac{Mc}{\hbar}\right)^2\Phi = 0. \tag{13}$$

In the remainder of this paper, I derive the Lagrangian densities providing corrections to the known quantum fields, the Dirac, KG, Proca, and Higgs. The basis for these corrections is Eq. (12) providing some interesting consequences including a spontaneous transformation between mass and charge.

IV. SPIN ½ FERMION

A. Dirac Lagrangian

A Dirac Lagrangian may be written as

$$\mathscr{L} = i\hbar c\bar{\Psi}\gamma^\mu\partial_\mu\Psi - Mc^2\bar{\Psi}\Psi. \tag{14}$$

The Euler–Lagrange equations for the above are

$$\begin{aligned}&\text{EL1: } i\hbar\gamma^\mu\partial_\mu\Psi - Mc\Psi = 0, \text{ and}\\ &\text{EL2: } i\hbar(\partial_\mu\overline{\Psi})\gamma^\mu + Mc\overline{\Psi} = 0.\end{aligned} \tag{15}$$

The two Euler–Lagrange equations represent a particle and its antiparticle, respectively.

A local U(1) gauge transformation may be performed to Eq. (14) and a term representing photons may be added without affecting the gauge invariance. The result is

$$\begin{aligned}\mathscr{L} &= i\hbar c\bar{\Psi}\gamma^\mu\partial_\mu\Psi - Mc^2\bar{\Psi}\Psi\\ &\quad - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - \left(q\bar{\Psi}\gamma^\mu\Psi\right)A_\mu\,,\end{aligned} \tag{16}$$

which then yields three Euler–Lagrange equations

$$\begin{aligned}&\text{EL1: } i\hbar\gamma^\mu\partial_\mu\Psi - Mc\Psi - q\gamma^\mu\Psi A_\mu = 0,\\ &\text{EL2: } i\hbar\partial_\mu\overline{\Psi}\gamma^\mu + Mc\overline{\Psi} + q\gamma^\mu\Psi A_\mu = 0,\\ &\text{EL3: } \frac{1}{4\pi}\partial_\mu F^{\mu\nu} - q\bar{\Psi}\gamma^\mu\Psi = 0.\end{aligned} \tag{17}$$

B. Modified Dirac Lagrangian

The MKG equation, Eq. (12), may be decoupled into the bispinor equations by deploying the Dirac formalism.[10] This was done in the author's previous work[1] and here I only state the result.

We define I (or simply 1) to be a 2×2 unit matrix, and σ^i to be 2×2 Pauli matrices, γ^0 to be the first of the 4x4 Dirac matrices (the others are γ^i, $i = 1,2,3.$). By using the first of the following relationships:

$$\begin{aligned}\gamma^0 - 1 &= 2\begin{pmatrix}0 & 0\\ 0 & -1\end{pmatrix},\\ \gamma^0 + 1 &= 2\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix},\end{aligned} \tag{18}$$

the quantum wave equation describing the spin half fermion may be written

$$i\hbar\gamma^\mu\partial_\mu\Psi + \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\Psi = 0. \tag{19}$$

This is the kinetic energy-operated, modified Dirac equation compared to the Dirac equation, EL1 of Eq. (15).

A Lagrangian for the modified Dirac equation, Eq. (19), may be constructed as

$$\mathscr{L} = i\hbar c\bar{\Psi}\gamma^\mu\partial_\mu\Psi + \left(\gamma^0 - 1\right)\frac{Mc^2}{\gamma}\bar{\Psi}\Psi \tag{20}$$

of which the Euler–Lagrange equations are

$$\begin{aligned}&\text{EL1: } i\hbar\gamma^\mu\partial_\mu\Psi + \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\Psi = 0, \text{ and}\\ &\text{EL2: } -i\hbar(\partial_\mu\overline{\Psi})\gamma^\mu + \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\overline{\Psi} = 0.\end{aligned} \tag{21}$$

The two Euler–Lagrange equations represent a particle and its antiparticle, respectively. The Euler–Lagrange equations, Eq. (21), of the modified Dirac Lagrangian closely match those of the Dirac Lagrangian, Eq. (15), a critical difference being each of Eq. (21) includes both the massive and massless interaction between spinors. The EL1 of Eq. (21), or Eq. (19), may be solved for the particles at rest, with the results

$$\begin{aligned}\Psi_A &= \text{constant},\\ \Psi_B &= e^{\pm i\frac{Mc^2}{\hbar/2}t}\Psi_B(0).\end{aligned} \tag{22}$$

In addition to the solutions representing the particle and antiparticle pair (Ψ_B in the above, although Ψ_A and Ψ_B may be interchanged), the massless interaction leads to a constant solution which can be set to zero representing a true vacuum state. This was discussed in Ref. 1. The lack of this solution by the Dirac equation, Eq. (15), unfortunately has been the source of the difficulty (e.g., "Dirac sea"[11]) of an otherwise extremely successful approach by Dirac. In QED, the electron and positron are on equal footing but still no true vacuum state exists. The vacuum state solution of Eq. (22) appears to finally remove this difficulty, one of the remarkable accomplishments of the present approach.

The modified Dirac Lagrangian, Eq. (20), may be gauge-transformed to

$$\begin{aligned}\mathscr{L} &= i\hbar c\bar{\Psi}\gamma^\mu\partial_\mu\Psi + \left(\gamma^0 - 1\right)\frac{Mc^2}{\gamma}\bar{\Psi}\Psi\\ &\quad - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - \left(q\bar{\Psi}\gamma^\mu\Psi\right)A_\mu,\end{aligned} \tag{23}$$

which then yields three Euler–Lagrange equations

$$\begin{aligned}&\text{EL1: } i\hbar\gamma^\mu\partial_\mu\Psi + \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\Psi - q\gamma^\mu\Psi A_\mu = 0,\\ &\text{EL2: } i\hbar\partial_\mu\overline{\Psi}\gamma^\mu - \left(\gamma^0 - 1\right)\frac{Mc}{\gamma}\overline{\Psi} + q\overline{\Psi}\gamma^\mu A_\mu = 0,\\ &\text{EL3: } \frac{1}{4\pi}\partial_\mu F^{\mu\nu} - q\bar{\Psi}\gamma^\mu\Psi = 0.\end{aligned} \tag{24}$$

The Euler–Lagrange equations, Eq. (24), of the modified Dirac Lagrangian closely match those of the Dirac Lagrangian, Eq. (17), with a critical difference being that the first two of Eq. (24) include both the massive and massless interactions between spinors. The gauge fields (EL3 of each) are exactly the same.

V. SCALAR BOSON

A. The Klein–Gordon field

Now let $\Phi = \phi_1 + i\phi_2$ and $\Phi^* = \phi_1 - i\phi_2$ where ϕ_1 and ϕ_2 are two real fields. We note that the Lagrangian for the KG equation, Eq. (13), for the complex-valued scalar fields may be written

$$\mathscr{L} = \left(\partial_\mu\Phi^*\right)\left(\partial^\mu\Phi\right) - \left(\frac{Mc}{\hbar}\right)^2\Phi^*\Phi. \tag{25}$$

By treating Φ^* and Φ to be independent variables, one obtains the Euler–Lagrangian equations

$$\begin{aligned} &\text{EL1: } \partial_\mu \partial^\mu \Phi + \left(\frac{Mc}{\hbar}\right)^2 \Phi = 0, \\ &\text{EL2: } \partial_\mu \partial^\mu \Phi^* + \left(\frac{Mc}{\hbar}\right)^2 \Phi^* = 0. \end{aligned} \tag{26}$$

A local U(1) gauge transformation via

$$\begin{aligned} &\mathscr{D}^\mu \rightarrow \partial^\mu + i\beta A^\mu \text{ for } \Phi, \\ &\mathscr{D}^\mu \rightarrow \partial^\mu - i\beta B^\mu \text{ for } \Phi^*, \end{aligned} \tag{27}$$

where

$$\beta \equiv \frac{q}{\hbar c}, \tag{28}$$

where q is the charge of the particle, and A^μ and B^μ are some vector fields associated with Φ and Φ^*, respectively, leads to the gauge transformed KG Lagrangian

$$\begin{aligned} \mathscr{L} = &\left[\left(\partial_\mu \Phi^*\right)\left(\partial^\mu \Phi\right) - \left(\frac{Mc}{\hbar}\right)^2 \Phi^* \Phi\right] \\ &+ \left[-\frac{i}{8\pi\hbar c} F^{\mu\nu} G_{\mu\nu} + \beta^2 (\Phi^*\Phi) B^\mu A_\mu\right] \\ &+ i\beta\left[\left(\partial_\mu \Phi^*\right)\Phi A^\mu - \Phi^*\left(\partial_\mu \Phi\right) B^\mu\right]. \end{aligned} \tag{29}$$

Note that we added a free field, $F^{\mu\nu}G_{\mu\nu}$, for four-vector fields, A^μ and B^μ, with their respective field strength tensor

$$\begin{aligned} F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu, \\ G^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu. \end{aligned} \tag{30}$$

Equation (29) includes the KG Lagrangian, Eq. (25), as expected and additional terms showing the interaction among the scalar fields and the vector fields. The mass term remains unchanged and this presents a problem in the weak interactions where a transformation between massive and massless gauge vector field is necessary.[12,13]

Since Φ and Φ^* are treated to be independent of each other, so are B_μ and A_μ. If $B_\mu = A_\mu$, then Eq. (27) specifies that Φ and Φ^* be made locally invariant in the opposite rotation, i.e., $\Phi \rightarrow e^{i\theta}\Phi$ and $\Phi^* \rightarrow e^{-i\theta}\Phi^*$, respectively, where θ is any real number. On the other hand, if $B_\mu = -A_\mu$, Eq. (27) specifies that both Φ and Φ^* be made locally invariant in the same rotation, i.e., by $\Phi \rightarrow e^{i\theta}\Phi$ and $\Phi^* \rightarrow e^{i\theta}\Phi^*$, respectively.

In the former case ($B_\mu = A_\mu$), by the use of Eq. (26) and denoting the conserved Noether current, $j_\mu = i\left[\left(\partial_\mu \Phi^*\right)\Phi - \Phi^*\left(\partial_\mu \Phi\right)\right]$ Eq. (29) may be written

$$\begin{aligned} \mathscr{L} = &\left[\left(\partial_\mu \Phi^*\right)\left(\partial^\mu \Phi\right) - \left(\frac{Mc}{\hbar}\right)^2 \Phi^* \Phi\right] \\ &+ \left[-\frac{i}{8\pi\hbar c} F^{\mu\nu} F_{\mu\nu} + \beta^2 (\Phi^*\Phi) \mathrm{A}^\mu \mathrm{A}_\mu + \beta j^\mu A_\mu\right]. \end{aligned} \tag{31}$$

The first bracket reproduces Eq. (25). The second bracket includes terms related to the Proca and Maxwell equations but remains massless.

In the latter case ($B_\mu = -A_\mu$), since $\left(\partial_\mu \Phi^*\right)\Phi + \Phi^*\left(\partial_\mu \Phi\right) = \partial_\mu(\Phi^*\Phi)$ Eq. (29) becomes

$$\begin{aligned} \mathscr{L} = &\left[\left(\partial_\mu \Phi^*\right)\left(\partial^\mu \Phi\right) - \left(\frac{Mc}{\hbar}\right)^2 \Phi^* \Phi\right] \\ &+ \left[\frac{i}{8\pi\hbar c} F^{\mu\nu} F_{\mu\nu} + \beta^2 (\Phi^*\Phi) \mathrm{A}^\mu \mathrm{A}_\mu + i\beta \partial_\mu (\Phi^*\Phi) A^\mu\right]. \end{aligned} \tag{32}$$

The second bracket remains massless. If $\partial_\mu(\Phi^*\Phi)$ vanishes, then $\Phi^*\Phi$ is constant and the second bracket further simplifies to look like the Proca equation with the mass term replaced by a current term. The gauge transformation of the KG Lagrangian fails to produce a massive gauge vector field. In the following, the gauge transformation of the MKG Lagrangian is shown to allow such transformation to occur without resorting to the Higgs field.

The above gauge-transformed KG field will be referenced in the following (Sec. V.B and Sec. VI) sections and compared to the gauge-transformed, MKG field.

B. The modified Klein–Gordon field

For the MKG equation, Eq. (12), one can write a Lagrangian for the complex-valued scalar fields,

$$\mathscr{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^*\left(\partial_\mu \Phi\right) \equiv \mathscr{L}_\alpha \tag{33}$$

of which the Euler–Lagrange equations are

$$\begin{aligned} &\text{EL1: } \partial_\mu \partial^\mu \Phi = i\alpha I_0^\mu \partial_\mu \Phi, \text{ and} \\ &\text{EL2: } \partial_\mu \partial^\mu \Phi^* = -i\alpha I_0^\mu \partial_\mu \Phi^*. \end{aligned} \tag{34}$$

EL1 in the above is the same as Eq. (12). EL2 represents its antiparticle. The MKG Lagrangian, Eq. (33), describes a massive, scalar, spin-zero boson with mass

$$M = \left(\frac{\gamma\alpha}{2}\right)\frac{\hbar}{c} \tag{35}$$

as back-calculated from Eq. (8) and carried by the term $\Phi^*(\partial_0 \Phi)$.

By a local U(1) gauge transformation for Eq. (33) via Eq. (27) and by inserting a free field, $F^{\mu\nu}G_{\mu\nu}$, we obtain a complete, gauge-transformed Lagrangian for the MKG equation

$$\begin{aligned} \mathscr{L} = &\left[\partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^*\left(\partial_\mu \Phi\right)\right] \\ &+ \frac{2i}{\hbar c}\left[-\frac{1}{16\pi} F^{\mu\nu} G_{\mu\nu} + i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu - \beta B^\mu) A_\mu\right] \\ &+ i\beta\left[(\partial^\mu \Phi^*)\Phi A_\mu - \Phi^*(\partial^\mu \Phi) B_\mu\right], \end{aligned} \tag{36}$$

where α and β represent mass and charge, respectively. Equation (36) combines a massive scalar field in the first square bracket, a massive vector field in the second square bracket, and the interaction of the scalar and the vector fields in the third square bracket.

From the second and the third term of Eq. (36), one can define a vector "potential" field

$$\mathscr{L} = -\frac{1}{16\pi}F^{\mu\nu}G_{\mu\nu} + i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu - \beta B^\mu)A_\mu + \frac{q}{2}\left[(\partial^\mu\Phi^*)\Phi A_\mu - \Phi^*(\partial^\mu\Phi)B_\mu\right]. \tag{37}$$

C. Massive and massless scalar bosons

It is interesting to note that when

$$\beta B^\mu = \alpha I_0^\mu, \tag{38}$$

i.e., $B^\mu = (2Mc^2/q\gamma, 0, 0, 0)$, (ignoring the free field) the second term of Eq. (36) vanishes and the first and the third terms merge so that it now reads

$$\mathscr{L} = \partial_\mu\Phi^*\partial^\mu\Phi + i\beta A^\mu(\partial_\mu\Phi^*)\Phi \equiv \mathscr{L}_\beta. \tag{39}$$

This represents a massless scalar boson with charge q in the vector field, A^μ. The Euler–Lagrange equations of the above are

$$\begin{aligned}&\text{EL1: } \partial_\mu\partial^\mu\Phi = -i\beta A^\mu\partial_\mu\Phi, \text{ and}\\ &\text{EL2: } \partial_\mu\partial^\mu\Phi^* = i\beta A^\mu\partial_\mu\Phi^*.\end{aligned} \tag{40}$$

Note that one can always select a particular B^μ independent of A^μ field equation without losing generality. Hence if I choose $B^\mu = (V, 0, 0, 0)$ with $V = 2Mc^2/q\gamma$, where V is a scalar potential, then the massive scalar boson, Eq. (33), transforms into a massless scalar boson, Eq. (39). I will bring γ to the left-hand side and rewrite this condition to note that γV is the γ-metric potential

$$\gamma V = \frac{2Mc^2}{q}. \tag{41}$$

Conversely, a massless scalar boson, Eq. (39) may be shown to transform to a massive scalar boson, Eq. (33) under certain conditions. For instance, if $B_\mu = A_\mu$ then a gauge transformation via

$$\begin{aligned}&\mathscr{D}^\mu \to \partial^\mu - i\alpha I_0^\mu \text{ for } \Phi,\\ &\mathscr{D}^\mu \to \partial^\mu + i\alpha I_0^\mu \text{ for } \Phi^*,\end{aligned} \tag{42}$$

under a condition similar to Eq. (38)

$$\beta A^\mu = \alpha I_0^\mu \tag{43}$$

fulfills such transformation. If further I choose $A^\mu = (V, 0, 0, 0)$ with V, a scalar potential, I then get

$$M = \left(\frac{\gamma V}{2}\right)\frac{q}{c^2} \tag{44}$$

and one can say the massless boson, Eq. (39), acquired mass M from charge q. Here, the scalar bosons transform between a massive state, Eq. (33), and a massless, charged state, Eq. (39), whenever the three-vector potential vanishes.

VI. VECTOR BOSON

A. Massless gauge vector field

We can obtain a massless gauge vector field from Eq. (37) under certain conditions: for instance, if the vector fields satisfy $B_\mu = A_\mu$, it then reads

$$\mathscr{L} = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu - \beta A^\mu)A_\mu + \frac{q}{2}[(\partial^\mu\Phi^*)\Phi - \Phi^*(\partial^\mu\Phi)]A_\mu. \tag{45}$$

One can take the divergence of the square bracket term and use Eq. (34) to find

$$\partial_\mu[(\partial^\mu\Phi^*)\Phi - \Phi^*(\partial^\mu\Phi)] = -i\alpha I_0^\mu\partial_\mu(\Phi^*\Phi), \tag{46}$$

hence within a constant

$$(\partial^\mu\Phi^*)\Phi - \Phi^*(\partial^\mu\Phi) = -i\alpha I_0^\mu(\Phi^*\Phi). \tag{47}$$

The third term of Eq. (45) then erases the mass (α) in the second term to result in

$$\mathscr{L} = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - i\frac{q}{2}\Phi^*\Phi\beta A^\mu A_\mu. \tag{48}$$

If in addition $\Phi^*\Phi = \phi_1^2 + \phi_2^2 = K \neq 0$(a nonzero constant), i.e., along a circle in $\phi_{1,}\phi_2$ plane, it can be absorbed by A^μ and Eq. (45) finally reduces to

$$\mathscr{L} = -\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - \frac{i}{2}q\beta A^\mu A_\mu. \tag{49}$$

This is a massless gauge boson field, or a modified massless Proca Lagrangian, of which the Euler–Lagrange equation is

$$\text{EL1: } \frac{1}{4\pi}\partial_\mu F^{\mu\nu} - iq\beta A^\nu = 0, \tag{50}$$

where $q\beta = q^2/(\hbar c)$.

With Eq. (49), the complete Lagrangian, Eq. (36), now reduces to

$$\mathscr{L} = \mathscr{L}_\alpha + \frac{2i}{\hbar c}\left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} - \frac{i}{2}q\beta A^\mu A_\mu\right]. \tag{51}$$

B. Massive gauge vector field

If $B_\mu = -A_\mu$, Eq. (37) reads

$$\begin{aligned}\mathscr{L} &= \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu + \beta A^\mu)A_\mu + \frac{q}{2}\partial^\mu(\Phi^*\Phi)A_\mu, \text{ or}\\ \mathscr{L} &= \frac{1}{16\pi}G^{\mu\nu}G_{\mu\nu} - i\frac{q}{2}\Phi^*\Phi(\alpha I_0^\mu - \beta B^\mu)B_\mu - \frac{q}{2}\partial^\mu(\Phi^*\Phi)B_\mu.\end{aligned} \tag{52}$$

The above simplifies if the scalar field satisfies $\partial^\mu(\Phi^*\Phi)=0$ and $\Phi^*\Phi=\phi_1^2+\phi_2^2=K\neq 0$ (a nonzero constant). In this case, K can be absorbed by the vector field and the above reduces to

$$\mathscr{L}=\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}+i\frac{q}{2}(\alpha I_0^\mu+\beta A^\mu)A_\mu,\ \text{or}\\ \mathscr{L}=\frac{1}{16\pi}G^{\mu\nu}G_{\mu\nu}-i\frac{q}{2}(\alpha I_0^\mu-\beta B^\mu)B_\mu, \tag{53}$$

which may be called modified Proca Lagrangians.

The Euler–Lagrange equations for Eq. (53)

$$\text{EL1: }\frac{1}{4\pi}\partial_\mu F^{\mu\nu}-iq\left(\frac{\alpha}{2}I_0^\nu+\beta A^\nu\right)=0,\\ \text{EL2: }\frac{1}{4\pi}\partial_\mu G^{\mu\nu}+iq\left(\frac{\alpha}{2}I_0^\nu-\beta B^\nu\right)=0, \tag{54}$$

are the modified Proca equations with a relativistic mass $M/\gamma=\alpha\hbar/(2c)$.

With the first of Eq. (53), the complete Lagrangian, Eq. (36), now reads

$$\mathscr{L}=\mathscr{L}_\alpha+\frac{2i}{\hbar c}\left[\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}+i\frac{q}{2}(\alpha I_0^\mu+\beta A^\mu)A_\mu\right]. \tag{55}$$

In contrast to Eq. (51), the above now includes a mass term in the gauge field and may be called a modified Higgs field. The first term, $\mathscr{L}_\alpha$, in the above then defines a particular scalar boson which may be called the modified Higgs boson.

C. Transformation between the massless and massive states of the gauge fields

It is interesting to note that the scalar field condition

$$\Phi^*\Phi=\phi_1^2+\phi_2^2=K \tag{56}$$

necessarily requires $\partial^\mu(\Phi^*\Phi)=0$ and define a circle in ϕ_1,ϕ_2 plane. The massless vector field, Eq. (49), and massive vector field, Eq. (53), both arise when $K\neq 0$. This compares to the condition by which the Higgs boson field arises via the spontaneous symmetry breaking of the Mexican hat or wine bottle potential with stable local minima present along the circle.

In the present theory, the transformations between the massless state, Eq. (49), and massive state, Eq. (53), of the gauge boson occur between the two admissible states of local gauge invariance: (1) $B_\mu=A_\mu$, i.e., Φ and Φ^* be made locally invariant in the opposite rotation, $\Phi\to e^{i\theta}\Phi$ and $\Phi^*\to e^{-i\theta}\Phi^*$, respectively, and (2) $B_\mu=-A_\mu$, i.e., both Φ and Φ^* be made locally invariant in the same rotation, $\Phi\to e^{i\theta}\Phi$ and $\Phi^*\to e^{i\theta}\Phi^*$, respectively.

D. Comparison with the Higgs field

For comparisons, the Proca Lagrangian may be written as

$$\mathscr{L}=-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}+\frac{1}{8\pi}\left(\frac{Mc}{\hbar}\right)^2A^\nu A_\nu \tag{57}$$

with the Euler–Lagrange equation

$$\partial_\mu F^{\mu\nu}+\left(\frac{Mc}{\hbar}\right)^2A^\nu=0. \tag{58}$$

According to the Proca Lagrangian, Eq. (57), mass is carried by the quadratic term of the vector field A^ν and may be created by a mechanism known as Brout–Englert–Higgs (BEH) mechanism which creates the Higgs boson. For example, consider a Lagrangian with a self-interaction potential energy terms[12–14]

$$\mathscr{L}=\frac{1}{2}\partial_\mu\Phi^*\partial^\mu\Phi+\frac{1}{2}\mu^2\Phi^*\Phi-\frac{1}{4}\lambda^2(\Phi^*\Phi)^2, \tag{59}$$

where μ and λ are real constants. By defining $\eta\equiv\phi_1-\mu/\lambda$, a gauge transformed and spontaneously symmetry-broken version of the above may be written[12]

$$\begin{aligned}\mathscr{L}=&\left[\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta)-(\mu)^2\eta^2\right]\\ &+\left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}+\frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2A_\mu A^\mu\right]\\ &+\left\{\frac{\mu}{\lambda}\left(\frac{q}{\hbar c}\right)^2\eta A_\mu A^\mu+\frac{1}{2}\left(\frac{q}{\hbar c}\right)^2\eta^2A_\mu A^\mu\right.\\ &\left.-\lambda\mu\eta^3-\frac{1}{4}\lambda^2\eta^4\right\}+\left(\frac{\mu^2}{2\lambda}\right)^2,\end{aligned} \tag{60}$$

where the first square bracket represents the Higgs scalar boson field with mass

$$M_S=\left(\sqrt{2}\mu\right)\frac{\hbar}{c}, \tag{61}$$

and the second square bracket a gauge boson field with mass

$$M_A=\left(2\sqrt{\pi}\frac{\mu}{\lambda}\right)\frac{q}{c^2}. \tag{62}$$

Its Euler–Lagrange equations are

$$\text{EL1: }\left[\partial_\mu\partial^\mu\eta-\mu^2\eta\right]-\beta^2\left(\frac{\mu}{\lambda}+\eta\right)A_\mu A^\mu+3\lambda\mu\eta^2+\lambda^2\eta^3=0,\\ \text{EL2: }\left[-\frac{1}{4\pi}\partial_\mu F^{\mu\nu}-\left(\beta\frac{\mu}{\lambda}\right)^2A^\nu\right]-\beta^2\left(\frac{2\mu}{\lambda}+\eta^2\right)A^\nu=0. \tag{63}$$

The square bracket term of the above EL1 is a KG equation defining the Higgs boson. The square bracket term of the above EL2 is a Proca equation describing a massive gauge boson.

Equations (36) and (60) are similar in their mathematical structure, combining a massive scalar boson field and a massive gauge boson field. Remarkably, Eq. (36) includes these fields as a result of the local U(1) gauge transformation of the MKG equation, Eq. (33), directly without introducing an arbitrary symmetry breaking process via Eq. (59). The mass of the scalar boson given in Eq. (35) will be identical with that of Eq. (61) if

$$\frac{\gamma\alpha}{2} = \sqrt{2}\mu \tag{64}$$

and the mass of the gauge boson given in Eq. (44) will be identical with that of Eq. (62) if

$$\frac{\gamma V}{2} = 2\sqrt{\pi}\frac{\mu}{\lambda}. \tag{65}$$

The Higgs scalar boson has been found experimentally.[15–25] It is possible that the modified Higgs boson in Eq. (55) is identical to the Higgs boson in Eq. (60), even though we arrive at them in quite different ways. It is because only one Higgs scalar boson has been found so far experimentally, but it will be interesting to see if the modified scalar boson may also be found experimentally that is not identical to the Higgs boson. Finally, it should be noted that the present theory leaves open a possible presence of entirely different or multiple "modified Higgs bosons" since it does not require local minima of the potential for the mass to be "created."

VII. DISCUSSIONS

This section is added after the initial submission of the manuscript to address some of the reviewers' comments. In this section, "(M)KG" may stand for either "(modified) Klein–Gordon" or "the (modified) Klein–Gordon equation."

A. The Hamiltonian

This paper concerns the Lagrangian description of the elementary particles. The Hamiltonian description is particularly useful for the (second) quantization of the fields, and may be derived from the Lagrangian. For the MKG Lagrangian $\mathscr{L}$, Eq. (33), the canonical conjugate fields may be written

$$\pi(x) = \frac{\partial\mathscr{L}}{\partial(\partial^0\Phi)} = \partial_0\Phi^* + i\alpha\Phi^*,$$

$$\pi^*(x) = \frac{\partial\mathscr{L}}{\partial(\partial^0\Phi^*)} = \partial_0\Phi.$$

We consider Φ and π and their Hermitian adjoints, $\Phi^\dagger$ and $\pi^\dagger$, respectively, to be field operators and write the Hamiltonian for MKG (without showing the details of the calculation)

$$H = \int d^3r\mathcal{H}(\mathbf{r},t),$$

where $\boldsymbol{r} \equiv x_i$ and the Hamiltonian density is

$$\mathcal{H}(x) = \pi(\partial^0\Phi) + (\partial^0\Phi^\dagger)\pi^\dagger - \mathscr{L}$$
$$= \pi\pi^\dagger + (\nabla\Phi^\dagger)\cdot(\nabla\Phi) - i\alpha\Phi^\dagger\pi^\dagger.$$

The commutator for an observable Q (or the operator, $\Phi, \pi, \Phi^\dagger$, or $\pi^\dagger$) with the Hamiltonian may then be shown to take the expected form

$$[Q,H] = i\hbar\frac{\partial}{\partial t}Q \equiv i\hbar c\partial_0 Q.$$

B. Lorentz covariance

The presence of the first order time derivative, $\partial/\partial t$, puts time on an unequal footing in the MKG equation which then violates Lorentz covariance, similar to the Schrödinger equation. The modified Dirac equation which derives from MKG recovers covariance by virtue of the Dirac spinors. Both MKG and KG obey the relativistic energy-momentum relation, however, as MKG, Eq. (12), may be converted back to the kinetic energy–momentum relation, Eq. (3), which is equivalent to the total energy–momentum relation, Eq. (1).

MKG may be viewed as symmetry-broken form of KG. To see this, we write the Schrödinger equation for a free particle as (by separating the space and time variables)

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2M}\nabla^2\Psi(\mathbf{r},t).$$

Let $\Psi(\mathbf{r},t) = \Psi(\mathbf{r})e^{-i\frac{T}{\hbar}t}$; by carrying out the time derivatives we then get a time-independent Schrödinger equation

$$-\frac{\hbar^2}{2M}\nabla^2\Psi(\mathbf{r}) \equiv -\frac{\hbar^2}{2M}\partial_i\partial^i\Psi(\mathbf{r}) = T\Psi(\mathbf{r}).$$

For direct comparisons, the MKG, Eq. (6), may be written in γ-metric

$$i\hbar\frac{\partial}{\partial t}\Phi(\mathbf{r},t) = \frac{\hbar^2}{2M}\Box\Phi(\mathbf{r},t) \equiv \frac{\hbar^2}{2M}\left(\partial_0\partial^0 - \partial_i\partial^i\right)\Phi(\mathbf{r},t).$$

Let $\Phi(\mathbf{r},t) = \Phi(\mathbf{r})e^{-i\frac{T}{\hbar}t}$; by carrying out the time derivatives we then get

$$-\frac{\hbar^2}{2M}\partial_i\partial^i\Phi(\mathbf{r}) = \frac{1}{2Mc^2}\left(T^2 + 2Mc^2T\right)\Phi(\mathbf{r}) \text{ or}$$
$$-\frac{\hbar^2}{2M}\partial_i\partial^i\Phi(\mathbf{r}) = \frac{P^2}{2M}\Phi(\mathbf{r}),$$

where we used the kinetic energy-momentum equation, $T^2 + 2Mc^2T = P^2c^2$. For $v \ll c$, $T \cong \frac{P^2}{2M}$, hence the time-independent MKG equation reduces to the time-independent Schrödinger equation.

On the other hand, for KG, EL1 of Eq. (26)

$$\partial_\mu\partial^\mu\Phi(\boldsymbol{r},t) + \left(\frac{Mc}{\hbar}\right)^2\Phi(\boldsymbol{r},t) = 0,$$

let $\Phi(\mathbf{r},t) = \Phi(\mathbf{r})e^{-i\frac{E}{\hbar}t}$; by carrying out the time derivatives we then get

$$-\frac{E^2}{c^2\hbar^2}\Phi(\mathbf{r}) - \partial_i\partial^i\Phi(\mathbf{r}) + \frac{M^2c^2}{\hbar^2}\Phi(\mathbf{r}) = 0 \text{ or}$$
$$-\frac{\hbar^2}{2M}\partial_i\partial^i\Phi(\mathbf{r}) = \frac{P^2}{2M}\Phi(\mathbf{r}),$$

where we used the total energy-momentum equation, $E^2 - M^2c^4 = P^2c^2$. For $v \ll c$, this again reduces to the time-independent Schrödinger equation. Thus in the time-independent nonrelativistic limit, both KG and MKG reduce to the time-independent Schrödinger equation. Note, however, the assumed time-dependencies for KG and MKG are $e^{-i\frac{E}{\hbar}t}$ and $e^{-i\frac{T}{\hbar}t}(= e^{-i\frac{E-Mc^2}{\hbar}t} = e^{-i\frac{E}{\hbar}t}e^{i\frac{Mc^2}{\hbar}t})$, respectively. The relationship between the KG and MKG wave functions then is

$$\Phi_{MKG}(\mathbf{r},\mathrm{t}) = \Phi_{KG}(\mathbf{r},\mathrm{t})e^{i\frac{Mc^2}{\hbar}t}.$$

Let $\frac{Mc^2}{\hbar} = \omega$, an angular velocity, then $e^{i\frac{Mc^2}{\hbar}t} = e^{i\omega t} = e^{i\theta}$, where $\theta = \theta(t)$ is a function of time only. This makes the transformation from KG to MKG to break the space–time symmetry. A spontaneous symmetry breaking is already built into the MKG equation.

The Higgs mechanism introduces a symmetry breaking by an arbitrary Lagrangian (Higgs) field and then asserts the symmetry breaking to be a necessary condition for the creation of mass. As seen in the above, however, the mass is already in the system. It just caused a transformation that breaks the space–time symmetry.

C. Probability density of the modified Klein–Gordon wave function

To show the probability density of the MKG equation, recall the Euler–Lagrange equations, Eq. (34). Now Φ^* x EL1 $-\ \Phi$ x EL2 gives

$$\partial_\mu(\Phi^*\partial^\mu\Phi - \Phi\partial^\mu\Phi^*) = i\alpha\partial_0(\Phi^*\Phi),$$

which is just Eq. (46). In the bracket of the left hand side is the Noether current which will be conserved as long as $\Phi^*\Phi$ of the right hand side is a constant in time. Thus $\Phi^*\Phi$, being a positive definite constant, may be interpreted as the probability density (with suitable normalization). Similar calculations for the KG equation do not allow the probability density interpretation of the wave function. But it was shown in the above Section VII.B that MKG is a symmetry-broken version of the KG, hence the new theory (MKG) accommodates the old theory (KG) as a special case rather than contradicting it.

D. Renormalization

Renormalization is an essential feature of the quantum field theory. For instance, the amplitudes of a one-loop Feynman diagram in quantum electrodynamics involve logarithmic divergence roughly in the form

$$\int^\infty \frac{dP}{P} = \ln P|^\infty = \infty.$$

The procedure of removing the unobservable divergence from the upper end is called renormalization. Although extensively studied, it is still considered conceptually unsatisfactory.

The above, however, may be rewritten

$$\int^\infty \frac{dP}{P} = \int^\infty \frac{d(\gamma M v)}{\gamma M v} = \int^\infty \frac{d\gamma}{\gamma} + \int^c \frac{dv}{v} = \ln(\gamma)|^\infty + \ln(|v|)|^c,$$

where $\gamma \leq \infty$ and $v \leq c$. It is clear that the upper bound divergence is caused by the γ-integration. But in the $1/\gamma$ metric formulation, γ is embedded in the mass term represented by M/γ and the above integral would then show up as

$$\int^{Mc} \frac{d\mathscr{P}}{\mathscr{P}} = \int^{Mc} \frac{d(Mv)}{Mv} = \int^c \frac{dv}{v} = \ln(|v|)|^c + \mathrm{Const.}$$

The $1/\gamma$ metric formulation pays off by removing the γ-divergence a priori and rendering renormalization unnecessary, at least in the above sense. Further accounts of the effect of the $1/\gamma$ metric formulation to the renormalization procedure are warranted.

VIII. CONCLUSION

The KG equation is obtained by applying the quantum prescriptions to the momentum and to the total energy in the relativistic energy-momentum relation. When we apply a quantum prescription to the total energy that is the sum of the internal (rest) energy and the kinetic energy, we are attempting to describe the two different sources of energy by a single set of wave equations.

This paper describes a novel approach to resolve this fundamental problem. A kinetic energy-operation (instead of the total energy-operation) is used to obtain a MKG equation, from which the modified quantum fields are then derived: MKG field for a massive scalar boson, modified Dirac field for a spin half fermion, modified Proca field for a massive vector boson, and modified Higgs field for a massive scalar boson and massive gauge vector boson.

The main mathematical results are summarized in the Appendix. The modified Dirac field thus derived closely matches the Dirac field, a crucial difference being that former includes both the massive and massless interaction between spinors. The equations of motion then yield the particles-at-rest solutions that include a vacuum state. The original Dirac equation lacks this feature, which led Dirac to hypothesize existence of the Dirac sea. It is clear that this difficulty arose because the KG equation upon which Dirac equation is based upon uses the total energy as the basis of quantum prescription. This breakthrough by the new approach lends support to other results that follow.

The MKG field features scalar bosons in massive state and a massless, charged state transforming spontaneously to each other when the three-vector potential vanishes. The MKG Lagrangian is shown to yield both the modified Proca field and the modified Higgs field directly by a local U(1) gauge transformation.

The Higgs and modified Higgs fields are similar in their mathematical structure combining a massive scalar boson and a massive gauge boson. Remarkably, however, the modified Higgs field includes these bosons as a result of the local U(1) gauge transformation of the MKG Lagrangian without introducing an additional symmetry breaking process.

It is probable that the scalar bosons in both fields (Higgs and modified Higgs) are only the same, even though we arrive at them in quite different ways. It is because only one Higgs scalar boson has been found so far experimentally, but it will be interesting to see if the modified Higgs scalar boson may also be found experimentally that is not the same as the

Higgs boson. Finally, it should be noted that the present theory leaves open a possible presence of entirely different or multiple modified Higgs bosons since it does not require local minima of the potential for the mass to be created.

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APPENDIX: LAGRANGIAN DENSITY FOR QUANTUM FIELDS—STANDARD FORMULATION VERSUS MODIFIED

	Standard formulation	Modified
Massive scalar boson (Klein–Gordon versus modified)	$\mathscr{L} = (\partial_\mu \Phi^*)(\partial^\mu \Phi) - \left(\frac{Mc}{\hbar}\right)^2 \Phi^*\Phi$	$\mathscr{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^* (\partial_\mu \Phi); \alpha \equiv \frac{2Mc}{\hbar\gamma}$
Massless scalar boson (Goldstone versus modified)	$\mathscr{L} = \partial_\mu \Phi^* \partial^\mu \Phi$	$\mathscr{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\beta A^\mu (\partial_\mu \Phi^*)\Phi; \beta \equiv \frac{q}{\hbar c}$
Spin 1/2 Fermion (Dirac versus modified)	$\mathscr{L} = i\hbar c \bar{\Psi}\gamma^\mu \partial_\mu \Psi - Mc^2 \bar{\Psi}\Psi$	$\mathscr{L} = i\hbar c \bar{\Psi}\gamma^\mu \partial_\mu \Psi + (\gamma^0 - 1)\frac{Mc^2}{\gamma}\bar{\Psi}\Psi$
Spin 1/2 Fermion + massless gauge field (Dirac versus modified)	$\mathscr{L} = i\hbar c \bar{\Psi}\gamma^\mu \partial_\mu \Psi - Mc^2 \bar{\Psi}\Psi - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - (q\bar{\Psi}\gamma^\mu \Psi) A_\mu$	$\mathscr{L} = i\hbar c \bar{\Psi}\gamma^\mu \partial_\mu \Psi + (\gamma^0 - 1)\frac{Mc^2}{\gamma}\bar{\Psi}\Psi - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - (q\bar{\Psi}\gamma^\mu \Psi) A_\mu$
Massive gauge boson (Proca versus modified)	$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{Mc}{\hbar}\right)^2 A^\nu A_\nu$	$\mathscr{L} = \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + i\frac{q}{2}(\alpha I_0^\mu + \beta A^\mu) A_\mu$
Massless gauge boson (Massless Proca versus modified)	$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$	$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - i\frac{q}{2}\beta A^\mu A_\mu$
Higgs field (Φ^4) versus modified	$\mathscr{L} = \frac{1}{2}\partial_\mu \Phi^* \partial^\mu \Phi + \frac{1}{2}\mu^2 \Phi^*\Phi - \frac{1}{4}\lambda^2 (\Phi^*\Phi)^2$	$\mathscr{L} = \partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu \Phi^* (\partial_\mu \Phi)$
Massive scalar boson + massive gauge boson (Higgs versus modified)	$\mathscr{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - (\mu)^2 \eta^2\right] + \left[-\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right] + \left\{\frac{\mu}{\lambda}\left(\frac{q}{\hbar c}\right)^2 \eta A_\mu A^\mu + \frac{1}{2}\left(\frac{q}{\hbar c}\right)^2 \eta^2 A_\mu A^\mu - \lambda\mu\eta^3 - \frac{1}{4}\lambda^2\eta^4\right\} + \left(\frac{\mu^2}{2\lambda}\right)^2$	$\mathscr{L} = \left[\partial_\mu \Phi^* \partial^\mu \Phi + i\alpha I_0^\mu (\Phi^* \partial_\mu \Phi)\right] + \frac{2i}{\hbar c}\left[\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + i\frac{q}{2}(\alpha I_0^\mu + \beta A^\mu) A_\mu\right]$

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