The photon element

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(Received 27 March 2017; accepted 31 January 2018; published online 27 February 2018)

Abstract: According to Compton's own analysis, Compton scattering is an instantaneous collision between the photon of the incident gamma ray (or light) and the stationary electron where the photon has energy, the Planck constant multiplied by the frequency of light. In this article, I carry Compton's analysis farther to show it as much to be a collision in every cycle of the incident wave between the photon element and the moving electron, where the photon element has energy, the Planck constant per second. These two cases are shown to be substantially indistinguishable from each other owing to the relativistic effect. The photon element is then the true fundamental particle or the Planck element as it may be called for obvious reasons. The Planck element is likely to be the most fundamental quantum of the electromagnetic field. This new analysis provides a possible resolution for the wave-particle duality of light. © *2018 Physics Essays Publication.*
[http://dx.doi.org/10.4006/0836-1398-31.1.99]

Résumé: Selon l'analyse même de Compton, la diffusion de Compton est une collision instantanée entre le photon du rayon gamma incident (ou lumière) et l'électron stationnaire lorsque le photon possède de l'énergie, c-à-d. la constant de Planck multipliée par la fréquence de la lumière. Dans cet article, je développe l'analyse de Compton pour montrer qu'il s'agit autant d'une collision à chaque cycle de l'onde incidente entre le photon et électron en movement, où le photon possède de l'énergie, la constant de Planck par seconde. Ces deux cas se sont révélés difficiles à distinguer les uns des autres en raison de l'effet relativiste. Alors, le photon est la vraie particule fondamentale, ou l'élément de Planck, comme l'on peut l'appeler pour des raisons évidentes. L'élément de Planck est probablement le quantum le plus fondamental du champ électromagnétique. Cette nouvelle analyse fournit une résolution possible pour la dualité onde-particule de la lumière.

Key words: Light; Photon; Element; Quantum; Field.

I. INTRODUCTION

A photon is an elementary particle with energy $E = h\nu$ (h is the Planck constant and ν is the frequency), which always travels with the speed of light (or electromagnetic radiation in general), c. As per this view, all its energy is kinetic and its mass is zero. The wave aspects of light, however, cannot be thrown away as it manifests the wave properties, for instance, the single source (self) or the double-slit interference. A photon is then described as the quantum of an electromagnetic field, massless gauge boson with spin one, as per the quantum field theory of the Standard model. Regardless, it is difficult to imagine an elementary particle, a kind of black box with hidden structures yet to be elucidated, with the energy content not being constant but proportional to the frequency that is a property of its very wave nature.

II. THE PARTICLE BEHAVIOR OF LIGHT—COMPTON SCATTERING

A. Compton's demonstration of particle nature for light

Einstein introduced the particle nature of light in his Nobel-prize winning paper for photoelectricity in 1905.[1] In 1923, Compton further demonstrated it[2] by successfully explaining what is now called the Compton scattering by assuming a photon with energy $E = h\nu$ to be a single, elementary particle. This view of the particle nature of light has dominated our physics since.

Both the QED and the original calculation by Compton produce the relationship between the wavelengths of the incipient X-ray before and after the scattering

$$\lambda' - \lambda = \frac{h}{M_e c}(1 - \cos\theta), \tag{1}$$

or by the corresponding frequency relationship

$$\frac{\nu'}{\nu} = \frac{1}{1 + \alpha(1 - \cos\theta)}, \tag{2}$$

where λ and ν are the wavelength and frequency of the incident primary X-rays (or γ-rays), respectively, λ' and ν' are the wavelength and frequency of the scattered X-rays, respectively, M_e is the mass of the electron, θ is the scattering angle, and

$$\alpha \equiv \frac{h\nu}{M_e c^2}.$$

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The above equations may be derived from the conservation of energy and momentum if both the incoming X-ray photon and the stationary electron are single particles. Since the energy of the incident waves is $h\nu$ and the energy of the scattered wave is $h\nu'$, with $h\nu \geq h\nu'$, the problem is an inelastic scattering by the elastic collision between the photon and the electron with the photon losing some energy. It is noted that for a given direction, θ, the change in wavelength

$$\Delta\lambda = \lambda' - \lambda$$

is constant and does not depend on the frequency of the incident waves. For $\theta = 90°$, it is given by

$$\Delta\lambda(\theta = 90°) = \lambda_c = \frac{h}{M_e c} = 2.43 \times 10^{-12}\ \text{m},$$

where λ_c is known as the Compton wavelength. Since it is constant, the change in wavelength is experimentally noticeable only when it is not negligible compared to the wavelength of the incident electromagnetic waves.

Assuming that the electron is at rest initially, the speed ratio of the electron, $\beta' \equiv u'_e/c$, after the scattering is given by

$$\beta' = 2\alpha \sin\frac{\theta}{2}\frac{\sqrt{1+(2\alpha+\alpha^2)\sin^2\frac{\theta}{2}}}{1+2\alpha(1+\alpha)\sin^2\frac{\theta}{2}}. \tag{3}$$

B. The photon element model

In terms of the photon element, i.e., that with the angular momentum h or equivalently with the energy h/s (s is the time unit, second) and hereinafter called the "Planck element," the increased wavelength, given by Eq. (1), may be seen from the conservation of momentum of an electron and the incident X-ray Planck element before and after the collision. This geometric consideration is done as first approximation; more rigorous formulation of the model will then follow.

Figure 1 illustrates the travel of Planck elements, P1–P5 as examples, in the x-direction. Another way of thinking of Planck elements is that each wave of one full cycle produces one Planck element when the wave hits an object like an electron, more in line with the Copenhagen interpretation of quantum mechanics.

The distance between the adjacent Planck elements of the incident X-ray is the wavelength, λ. At time $t = 0$, the leading Planck element P1 collides with the electron and scatters at an angle θ with respect to the x-direction. The collision causes the electron to move by the amount with the x-component $\Delta\lambda$ before P2 arrives at the electron at $t = \lambda'/c$ rather than $t = \lambda/c$. After scattering, P1 is followed by P2 with λ' rather than λ distance apart. This is a geometric constraint for the change in the wavelength.

I now need to express the x-velocity of the electron, u_{ex}, in terms of the properties of the incident and scattered waves. Referring to Fig. 1, the electron recoils by the distance with

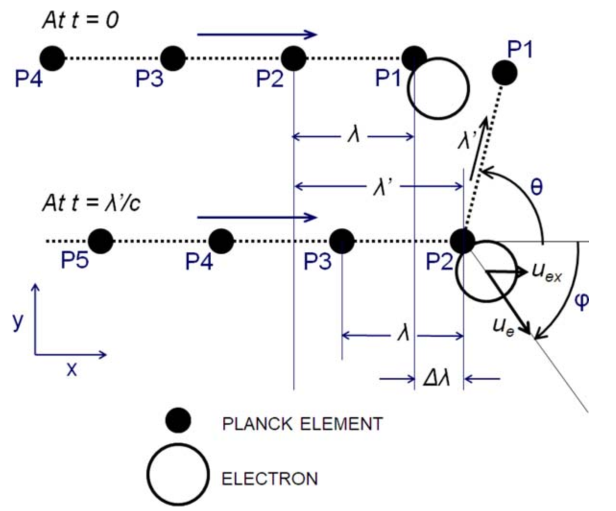

FIG. 1. (Color online) Illustration of the Compton scattering (not to scale).

the x-component $\Delta\lambda \equiv (\lambda' - \lambda)$ during the period of $1/\nu_0$ s. The electron moves by $\Delta\lambda$ every cycle of the incident wave, and hence, the total movement in 1 s by the electron must be $\nu_0\Delta\lambda$.

Hence, the x-velocity of the electron, u_{ex}, is

$$u_{ex} = \Delta\lambda\frac{\nu_0}{s} = \Delta\lambda\nu \tag{4}$$

or for every cycle, the x-velocity increment is

$$\Delta u_{ex} = \frac{u_{ex}}{\nu_0} = \frac{\Delta\lambda}{s}. \tag{5}$$

The X-ray Planck elements impinge upon an electron in the positive x-direction and scatter in the direction θ, with θ being the angle of the scattered beam with respect to the positive x-direction. The wavelength of the primary beam (i.e., before the collision) is λ, and that of the scattered beam (i.e., after the collision) is λ'. The speed of the Planck elements must be c both before and after the collision. By the classical two-body problem, the x-component of the momentum imparted to the electron by a Planck element that scatters at an angle θ can be shown to be

$$P_{px} = M_p c. \tag{6}$$

The same Planck element after the collision with the electron has the momentum in the θ direction

$$P_{p\theta} = M_p c. \tag{7}$$

Notice that the mass of the Planck element is unchanged and the effective speed of the Planck element is always the speed of light. The conservation of momentum for the two-body problem in the x-direction requires

$$M_e\Delta u_{ex} = M_p c(1 - \cos\theta). \tag{8}$$

With $M_p = \frac{h}{c^2}\frac{1}{s}$ and from Eqs. (5) and (8), I then find

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{M_e c}(1 - \cos\theta). \tag{9}$$

Thus, I see that the Compton scattering occurs as first approximation by an elastic collision between the X-ray Planck element and the moving electron, one Planck element at a time.

I will now rigorously calculate the frequency and speed of the electron after the collision by considering both the relativistic momentum and the energy conservation. I denote the initial velocity of the electron to be u_e and the velocity after the collision to be u_e' and define

$$\gamma = \sqrt{\frac{1}{1-\beta^2}}; \quad \beta = \frac{u_e}{c} \tag{10}$$

and

$$\gamma' = \sqrt{\frac{1}{1-\beta'^2}}; \quad \beta' = \frac{u_e'}{c}. \tag{11}$$

I will focus on the case, $\theta = \pi$, where the momentum conservation is given by

$$-\frac{h\nu'}{c} + \gamma' M_e \beta' c = \frac{h\nu}{c} + \gamma M_e \beta c, \tag{12}$$

while the energy conservation is given by

$$h\nu' + (\gamma' - 1)M_e c^2 = h\nu + (\gamma - 1)M_e c^2. \tag{13}$$

By using

$$h\nu \equiv M_p c^2 \nu_0 \tag{14}$$

and

$$h\nu' \equiv M_p c^2 \nu_0', \tag{15}$$

where

$$\nu_0 = \nu s; \quad \nu_0' = \nu' s,$$

Equations (12) and (13) then may be written in the forms appropriate for the Planck element model as follows:

$$-M_p \nu_0' + \gamma' M_e \beta' = M_p \nu_0 + \gamma M_e \beta \tag{16}$$

and

$$M_p \nu_0' + (\gamma' - 1)M_e = M_p \nu_0 + (\gamma - 1)M_e. \tag{17}$$

Equations (16) and (17) are completely equivalent to Eqs. (12) and (13).

Equations (12) and (13) may be solved for ν' and β'. After somewhat tedious calculation, I get

$$\frac{\nu'}{\nu} = \frac{1-\beta}{1+\beta+2\alpha\sqrt{1-\beta^2}}. \tag{18}$$

If $\beta = 0$, Eq. (18) reduces to Eq. (2) for $\theta = \pi$ as it should. As $\beta \to 1$, however, the first term quickly dominates and

$$\frac{\nu'}{\nu} \approx \frac{1-\beta}{1+\beta}. \tag{19}$$

The wavelength change corresponding to Eq. (18) is

$$\lambda' - \lambda = \frac{2\left(\beta + \alpha\sqrt{1-\beta^2}\right)}{1-\beta}\lambda. \tag{20}$$

The above $\lambda' - \lambda$ explicitly includes α and is convenient for the numerical calculation that follows. It may also be rearranged to read

$$\lambda' - \lambda = \frac{2\beta}{1-\beta}\lambda + \frac{2h}{M_e c}\sqrt{\frac{1+\beta}{1-\beta}}. \tag{21}$$

If $\beta = 0$, Eq. (21) reduces to Eq. (1) for $\theta = \pi$ as it should and it is easy to see as $\beta \to 1$

$$\lambda' - \lambda \approx \frac{2\beta}{1-\beta}\lambda. \tag{22}$$

I also get

$$\beta' = \frac{\sqrt{\beta^2 + 4\alpha^4(1-\beta)^2 + 8\alpha^3\sqrt{1-\beta^2}(1-\beta) + 4\alpha^2(1-\beta)(1+2\beta) + 4\alpha\beta\sqrt{1-\beta^2}}}{1 + 2\alpha\sqrt{1-\beta^2} + 2\alpha^2(1-\beta)}. \tag{23}$$

For convenience, I may write the above as

$$\beta' = f(\beta, \alpha), \tag{24}$$

where the function f is defined by the right-hand side of Eq. (23).

If $\beta = 0$, the above reduces to

$$\beta' = \frac{2\alpha(1+\alpha)}{1+2\alpha+2\alpha^2} \tag{25}$$

or to Eq. (3) for the case, $\theta = \pi$ with the electron initially at rest. For the Photon element model, the collision of a photon with the electron means repeating the collision of the Planck elements with the electron ν_0 times with α replaced with α/ν_0, beginning with $\beta_0 = 0$. By replacing β' with β_i and β with β_{i-1}, I may express Eq. (23) as

$$\beta_i = f\left(\beta_{i-1}, \frac{\alpha}{\nu_0}\right); \quad i = 1, \nu_0. \tag{26}$$

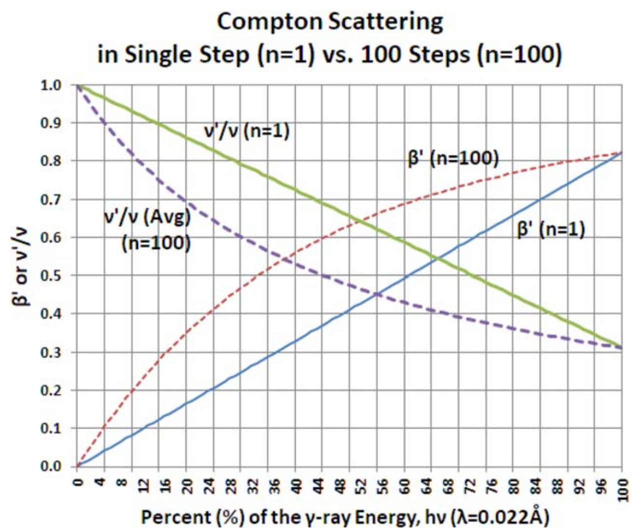

FIG. 2. (Color online) The speed of the electron over the speed of light and the energy ratio of the scattered γ-ray over that of the incident γ-ray are calculated for 1 step (as in Compton) and arbitrarily chosen 100 step compound collision. The results, $\beta_{100} \approx 0.82$ and $\frac{\nu_{100}}{\nu}$ (Average) ≈ 0.31, exactly match the results of the single particle solutions ($n = 1$) even though the changes are gradual and nonlinear for the former.

Since ν_0 is typically a large number, for instance, for an X-ray with $\lambda = 0.022$Å and $\nu_0 = 1.36 \times 10^{20}$, it will be impractical to numerically perform the compounded calculation for this large number. Instead, our strategy will be to compound the same calculations only manageable n times instead of ν_0 times, as follows:

$$\beta_i = f\left(\beta_{i-1}, \frac{\alpha}{n}\right); \quad i = 1, n. \tag{27}$$

Computationally, I can let n go to an arbitrary number, say $n = 100$. The results for $n = 1$ (which is for the case of a single particle with the energy, $h\nu$) and $n = 100$ are shown in Fig. 2. Although the latter case takes a gradual nonlinear path, the final value at $i = 100$ is the same as the single particle collision. I can conclude that the final compounded speed of the electron is the same for all n, i.e.,

$$\beta_n = f(0, \alpha) = f\left(\beta_{n-1}, \frac{\alpha}{n}\right) \approx 0.82 \text{ for all } n. \tag{28}$$

Similarly, by replacing ν' with ν_i and β with β_{i-1}, I may express the frequency ratio, or energy ratio, Eq. (18), as

$$\frac{E_i}{E} = \frac{\nu_i}{\nu} = g\left(\beta_{i-1}, \frac{\alpha}{\nu_0}\right); \quad i = 1, \nu_0. \tag{29}$$

Again, with ν_0 replaced with n, we get

$$\frac{E_i}{E} = \frac{\nu_i}{\nu} = g\left(\beta_{i-1}, \frac{\alpha}{n}\right); \quad i = 1, n. \tag{30}$$

For $n = 1$,

$$\frac{E_1}{E} = \frac{\nu_1}{\nu} = g(0, \alpha) = \frac{1}{1 + 2\alpha} \approx 0.31.$$

For $n = 100$,

$$\frac{E_{100}}{E} = \frac{\nu_{100}}{\nu} = g\left(\beta_{99}, \frac{\alpha}{100}\right) \approx \frac{1 - \beta}{1 + \beta} \approx 0.097.$$

But the energy ratio does not compound. Instead, for any nth step, one will be able to measure the average value of the frequency ratio, or the energy ratio, Eq. (30), to be

$$\frac{E_j}{E}(\text{Avg}) = \frac{\nu_j}{\nu}(\text{Avg}) = \left[\sum_{i=1,j} g\left(\beta_{i-1}, \frac{\alpha}{n}\right)\right] \Big/ j; \quad j = 1, n. \tag{31}$$

For $n = 1$

$$\frac{E_1}{E} = \frac{\nu_1}{\nu} \approx 0.31.$$

For $n = 100$, see Fig. 2 for the numerical calculation which shows

$$\frac{E_{100}}{E}(\text{Avg}) = \frac{\nu_{100}}{\nu}(\text{Avg}) \approx 0.31.$$

I can then conclude that the average value of the final energy ratio is the same for all n, i.e.,

$$\frac{E_n}{E}(\text{Avg}) = \frac{\nu_n}{\nu}(\text{Avg}) \approx 0.31 \text{ for all } n.$$

This result indicates that the scattering of a hypothetical single particle, the photon, is indistinguishable from the compound scattering of the ν_o number of Planck elements by the energy change of the incident γ-ray or by the final speed of the electron. This is a relativistic effect.

Let us also consider a multiple photon collision, which may be expressed by extending Eqs. (27) and (30), respectively, as

$$\beta_i = f\left(\beta_{i-1}, \frac{\alpha}{n}\right); \quad i = 1, kn; \; k = 1, \infty \tag{32}$$

and

$$\frac{E_i}{E} = \frac{\nu_i}{\nu} = g\left(\beta_{i-1}, \frac{\alpha}{n}\right); \quad i = 1, kn; \; k = 1, \infty, \tag{33}$$

where k represents the multiples of $h\nu$, the γ-ray energy. We can take an average value of the energy ratio by extending Eq. (31) for any integer k where $1 \le k \le \infty$ as

$$\frac{E_j}{E}(\text{Avg}) = \frac{\nu_j}{\nu}(\text{Avg}) = \left[\sum_{i=1,j} g\left(\beta_{i-1}, \frac{\alpha}{n}\right)\right] \Big/ j; \quad j = 1, kn; \; k = 1, \infty. \tag{34}$$

Multiple collisions increase the speed of the electron beyond the Compton scattering. Figure 3 shows a multiple collision for $n = 1$ and 10 and for $k = 6$. It may be seen that the value of β' rapidly converges to unity by multiple collisions. This

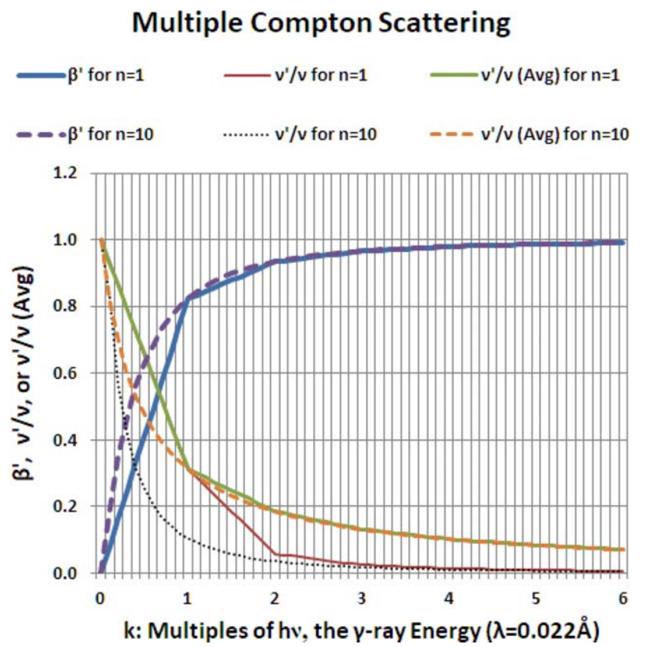

FIG. 3. (Color online) The speed of the electron for $\theta = \pi$ rapidly converges to the speed of light by multiple $h\nu$ collisions.

convergence is also seen by Eq. (23) which shows the convergence of β' to unity when $\beta' \sim \beta$.

The multiple scattering results further indicate that the scattering of a hypothetical single particle, the photon, is indistinguishable from the compound scattering of the ν_0 number of Planck elements by the energy change of the incident γ-ray or by the final speed of the electron.

III. SUMMARY AND CONCLUDING REMARKS

A photon with energy, $h\nu$, may now be understood to be a series of photon elements each having energy, h/s. It has been shown in this paper that the Compton scattering is explained equally well by this photon model as by the single photon particle of energy, $h\nu$, owing to the relativistic effect. The results of the elemental model of the photon are found to be almost indistinguishable from those of the single particle model. This explains how the single particle hypothesis, the "photon," has entered into our physics in spite of lacking any structure to visualize. This allows the elemental model to accommodate the concept of photon instead of contradicting it.

The photon element model is consistent with the wave nature of light with energy, $h\nu$, and wavelength, c/ν. The new analysis, therefore, provides a possible resolution for the so-called wave-particle duality since the wave and particle properties are shown here to be no longer mutually exclusive. The photon then is still a useful concept even though it hides the structure of light.

The derivation of Eq. (23) was the key to this extended analysis and as such is a valuable addition in itself to the analysis of Compton experiment. The model provides a structure of the photon which otherwise is simply a black box of the energy, $h\nu$. The photon element, or Planck element as I like to call it, rather than the notional single particle photon may be the most fundamental quantum of the electromagnetic field.

The term Planck element is distinguished from the "Planck particle" which refers to the hypothetical particle associated with the conventional Planck units.

I should mention here the alternate interpretations of the Compton experiment as the reviewer(s) of this manuscript pointed out. A "semiclassical" analysis describing the incoming X-ray as classical electromagnetic waves and the electron as a quantum mechanical particle with de Broglie matter-wave properties has been shown to successfully explain the Compton experiment.[3,4] A recent study[5] even suggests the concept of photon to be superfluous. The present analysis may be the first to indicate that the wave and particle properties are not mutually exclusive, but the former (the photon elements with the frequency ν) may appear to an observer as the latter, a result of the relativistic effect.

ACKNOWLEDGMENTS

The author thanks Dr. Art Liberman and Dr. Frank Crossman for the valuable discussions on the aspects of Compton scattering manifested in Gamma ray spectroscopy data. The author also thanks the anonymous reviewer(s) for bringing his attention to the work supporting the alternative interpretation of the Compton experiment.

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