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Relativistic dynamics for the expanding universe

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Abstract An analysis according to the principles of special and general relativity and less restrictive Newtonian gravity proves the dynamic effects to be substantial for the expanding universe. With the resulting dynamic critical density, typically greater than the standard critical density, I am able to identify the hypothetical cold dark matter (CDM) as being an artifact of the Friedmann–Robertson–Walker equation that is insufficient to describe the dynamic effects. With the included special-relativistic dynamic effects, I can now predict the cosmic data with two parameters, matter and the cosmological constant, without the CDM at least on a large scale.
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Résumé: Une analyse selon les principes de la relativité restreinte et générale et de la gravité newtonienne moins restrictive prouve que les effets dynamiques sont substantiels pour l'univers en expansion. Avec la densité critique dynamique résultante, généralement supérieure à la densité critique standard, je suis en mesure d'identifier l'hypothétique matière sombre froide comme étant un artefact de l'équation de Friedmann-Robertson-Walker qui est insuffisant pour décrire les effets dynamiques. Avec les effets dynamiques relativistes spéciaux inclus, je peux maintenant prédire les données cosmiques avec deux paramètres, la matière et la constante cosmologique, sans la matière sombre froide au moins à grande échelle.

Key words: Space; Time; Element; Gravity; Cosmological Constant; Dark Energy; Dark Matter; Special Relativity; General Relativity; Dynamics.

I. INTRODUCTION—THE STANDARD COSMOLOGICAL MODEL

The standard cosmological model describes the universe by the Friedmann (or Friedmann–Robertson–Walker, hereinafter FRW) equation[1–4]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_u - \kappa\frac{c^2}{a^2}, \tag{1}$$

where ρ_u is the average mass density of the universe, κ is a constant related to the curvature, and $a = a(t)$ is a dimensionless scale factor with the present epoch value, $a_0 = a(t_0)$, subscript 0 denoting the present (epoch) value. Note that Eq. (1) remains the same if substitution is made, $a \to a/\lambda$ and $\kappa \to \kappa/\lambda^2$, with both a and any parameter, λ, having the dimension of length. Henceforth in this work, we shall mean by $a(t)$ the radius of the universe unless specifically mentioned otherwise.

It is convenient to introduce the density parameters and the critical density of the present epoch

$$\Omega_i \equiv \rho_i/\rho_{\text{crit}}, \tag{2}$$

and

$$\rho_{\text{crit}} = 3H_0^2/(8\pi G), \tag{3}$$

respectively, where according to the ΛCDM model, i represents one of b, c, rad, κ, and Λ for baryon, cold dark matter (CDM), radiation, curvature, and the cosmological constant. The FRW equation may then be written

$$\left(\frac{H(a)}{H_0}\right)^2 = \Omega_m\left(\frac{a_0}{a}\right)^3 + \Omega_{\text{rad}}\left(\frac{a_0}{a}\right)^4 + \Omega_\kappa\left(\frac{a_0}{a}\right)^2 + \Omega_\Lambda, \tag{4}$$

where $H(a) = \dot{a}/a$ is the Hubble parameter with its present value, $H_0 = \dot{a}_0/a_0$, and $\Omega_m = \Omega_b + \Omega_c$. In the following, we shall ignore the radiation as its contribution is small. Later when we need to distinguish the critical density, Eq. (3), from the more general dynamic critical density (DCD), the former may be called the standard critical density (SCD) and denoted by putting a bar as in $\bar{\Omega}_i = \rho_i/\bar{\rho}_{\text{crit}}$ and $\bar{\rho}_{\text{crit}} = 3H_0^2/(8\pi G)$.

The FRW equation describes a slowly expanding universe as a nonrelativistic limit of the general theory of relativity (GR), derivable as well from the Newton's law of gravity. The observations show, however, the radial velocity of the mass content of the expanding universe gets faster with the increased distance from an observer until on the edge of the universe it nears the speed of the light, or even exceeds that according to the standard cosmological model. In this paper, I extend the FRW equation to include the effects of special-relativistic dynamics according to both the extended Newtonian gravity (Sections III–VI) and general relativity (Section VII). I then discuss the implications of these results for the expanding universe in Sections VIII and IX.

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II. AN ELEMENTAL SPACETIME, COSMOLOGICAL CONSTANT, AND DARK ENERGY

The author suggested the existence of photon elements in one[5] of his previous articles. This allowed one to define a system of fundamental units,[6] which then suggests that our spacetime is elemental. The photon element units derivable from the elemental spacetime have been shown to be compatible with Einstein's special theory of relativity (SR; hereinafter SR may also stand for special relativity or special-relativistic).

This further suggests that our space is filled with certain elements of the size of the photon element units and which is the medium of electromagnetic wave propagation. We shall call such elements "$\gamma-$elements" having the dimensions (on the order) of the photon elements units. We know a lot about the space that is filled with them. We call it the "vacuum," the medium for the electromagnetic wave propagation. It is the quantum field, because it is the only space we have. It must have the energy associated with the "cosmological constant," the only vacuum energy allowed to exist according to the GR at least in the universal sense.

We also know what it is not. It is not the "ether" space, an absolute space that is not allowed by the SR. The γ-element space is not an absolute space even though it is a medium of light propagation. Like any other matter, it must obey the special and general theories of relativity. Does the γ-element space violate the Michelson–Morley experiment? Below is a short account arguing why it does not.

The SR assumes from the outset (a) the principle of relativity and (b) constancy of the speed of light, c, in all inertial frames. Aharoni,[7] however, showed that (b) results from (a): i.e., a "signal" velocity common to all inertial frames of reference must exist as a result of the principal of relativity. An inertial frame that is stationary relative to the γ-element space is just one of the infinite numbers of the inertial frames. In this stationary inertial frame, the signal velocity must be the velocity of light. This velocity of light must then be the same in all inertial frames according to Aharoni's analysis. Michelson–Morley's experiment is consistent with this existence of the signal velocity and consequently consistent with the existence of the γ-element space. I do not need any additional analysis to conclude that the γ-element space does not violate the SR.

If we apply an Occam's razor to the γ-element space, in this paper one needs only one aspect of it: The nonzero mass of γ-elements allowing the Newton's two-body gravitational law applicable anywhere in the space. It also fits with the perfect-fluid model of materials that gives the stress-energy content, or the negative pressure, in the presence of the cosmological constant in GR. The name "γ-element" stems from the author's original conjecture[6] that the size of such fundamental element, if exist, should manifest itself as the wavelength of the highest energy electromagnetic waves or γ-ray[8–12] and should be observable.

By defining the mass density of γ-elements to be $\rho_\gamma = \rho_{\rm vac}/c^2$, where $\rho_{\rm vac}$ is the vacuum energy density, the cosmological constant, Λ, may then be expressed as[13,14]

$$\Lambda = \frac{8\pi G\rho_{\rm vac}}{c^4} = \frac{8\pi G\rho_\gamma}{c^2}, \tag{5}$$

where G is the gravitational constant. Λ has the dimension, $(1/{\rm m}^2)$. Thus, in this paper, I assume the mass density of the γ-elements, the cosmological constant, and the dark energy all refer the same thing in various units.

III. ASYMPTOTIC SEPARATION OF THE GRAVITATIONAL AND SPECIAL-RELATIVISTIC EFFECTS IN THE SCHWARZSCHILD METRIC

The Schwarzschild metric[15] may be written for a spherical body of mass, M, in the coordinates, (t, r, θ, ϕ) with $d\Omega^2 = d\theta^2 + \sin\theta^2 d\phi^2$,

$$c^2d\tau^2 = \left(1 - \frac{2GM}{c^2r}\right)c^2dt^2 - \left(1 - \frac{2GM}{c^2r}\right)^{-1}dr^2 - r^2d\Omega^2, \tag{6}$$

where τ is the proper time. The term in the bracket, $2GM/(c^2r)$, may be called a gravity factor; Eq. (6) reduces to the Minkowski spacetime metric if the gravity factor vanishes. To see the effect of a small gravity factor from the SR point of view, we consider a transformation of Eq. (6) from a coordinate system (t, r) to (t', r'). By ignoring the angular term and setting $c = 1$ (I will bring c back whenever more clarity is needed), the invariant of the transformation gives

$$d\tau^2 = \gamma_g^{-2}dt^2 - \gamma_g^2dr^2 = \gamma_g^{-2}dt'^2 - \gamma_g^2dr'^2, \tag{7}$$

where we define

$$\gamma_g \equiv \left(1 - \frac{2GM}{c^2r}\right)^{-1/2}. \tag{8}$$

Since the gravity factor is Lorentz scalar,[b)] γ_g is Lorentz scalar. We now look for the transformation of the type

$$dt' = \gamma_s(dt - \beta dr); \quad {\rm d}r' = \gamma_s(dr - \beta dt), \tag{9}$$

where $\beta = \dot{r}/c$ and γ_s is to be determined. By the use of Eq. (9), Eq. (7) may be written

$$\gamma_g^{-2}dt^2 - \gamma_g^2dr^2 = \gamma_g^{-2}\gamma_s^2(dt - \beta dr)^2 - \gamma_g^2\gamma_s^2(dr - \beta dt)^2. \tag{10}$$

By rearranging the terms, we can see this equation will hold approximately true if

[b)] We note that the gravity factor, $2GM/(c^2r) \equiv r_s/r$, where r_s is Schwarzschild radius, is a non-dimensional Lorentz scalar. Under a Lorentz transformation, both r_s and r contracts $r_s \to r_s/\gamma_s$ and $r \to r/\gamma_s$ where γ_s denotes the SR Lorentz factor, so the metric remains covariant. In a paper,[6] the author discussed the Lorentz covariance of units and universal constants which may be shown to be consistent with this result. The (length contracted) proper value of the Schwarzschild radius is then $r_s \to r_s/\gamma_s = 2GM/(\gamma_sc^2)$, suggesting for a photon, $r_s \to 0$. The implication warrants further investigation, especially regarding the black holes, but is beyond the scope of this paper.

$$\frac{1}{\gamma_s^2} = 1 - \gamma_g^4\beta^2 \approx 1 - \beta^2; \quad \gamma_g^4 \approx 1. \tag{11}$$

This shows the interaction of the gravity and the SR as the gravity factor becomes small, $r \gg r_s = 2GM/c^2$. Here, the transformation is asymptotically dominated by the SR and the Newton's gravitational law may be extended to include the SR effects by simply applying the SR Lorentz factor. It is as if the gravitation and the special relativity are independent of each other in an inertial frame.

IV. SPECIAL-RELATIVISTIC EXTENSION FOR THE NEWTON'S LAW OF GRAVITY

Consider the Newtonian gravitational law between two point bodies, denoted here as B_0 (where an observer is located, i.e., it is stationary) and B_1 (moving in the x direction with a velocity, v) having masses, M_0 and M_1, respectively, separated by the distance vector, $\underset{\sim}{r}(t)$. Equation (6) suggests the gravitational force between them at a given instant is[c]

$$\underset{\sim}{F} = G\frac{M_0 M_1}{r^2}\gamma^2\hat{e}_r; \text{ where } \gamma = \left[1 - (v/c)^2\right]^{-1/2}, \tag{12}$$

where $\hat{e}_r$ is the unit vector in the direction of $\underset{\sim}{r}(t)$, and v is the velocity of B_1 moving in the x direction. Since $\underset{\sim}{r}(t)$ is a function of time, the force vector is also a function of time. Equation (12) is equivalent to the interaction of B_0 with the gravitational potential of B_1

$$\Phi(r) = -\frac{GM_1}{r}\gamma; \quad \gamma = \left[1 - (v/c)^2\right]^{-1/2}. \tag{13}$$

For the expanding universe, every object outside B_0 moves away radially with no relative angular velocity, hence $v = \dot{r}$ and $\gamma = [1 - (\dot{r}/c)^2]^{-1/2}$. Furthermore, the universe in large scales is assumed to be isotropic and homogeneous. This allows an assumption of spherical symmetry with B_0 at the center, for example, to calculate the total gravitational force exerted by every mass of the universe by the use of the measured mass density of the universe, which is simply a collection of two-body problems summed or integrated. Equations (12) and (13) are the SR extension of the Newton's law of gravity and may be used to approximate the GR in case the distance between the two point bodies is much greater than the Schwarzschild radius $r \gg r_s$. The term special-relativistic also means dynamic and the two terms in many cases may be interchangeable.

V. AN ELEMENTAL SPACETIME MODEL OF THE UNIVERSE, TENSILE PRESSURE, AND THE EQUATIONS OF STATE

Let us consider a place in the γ-element space, where the gravity of no particular star or planet dominates. Here at a particular epoch a γ-element is assumed to interact with all masses of the universe gravitationally (see Fig. 1). We

[c]This is seen by the length contraction of r under SR or may be argued by more detailed discussion of Lorentz transformation.[6]

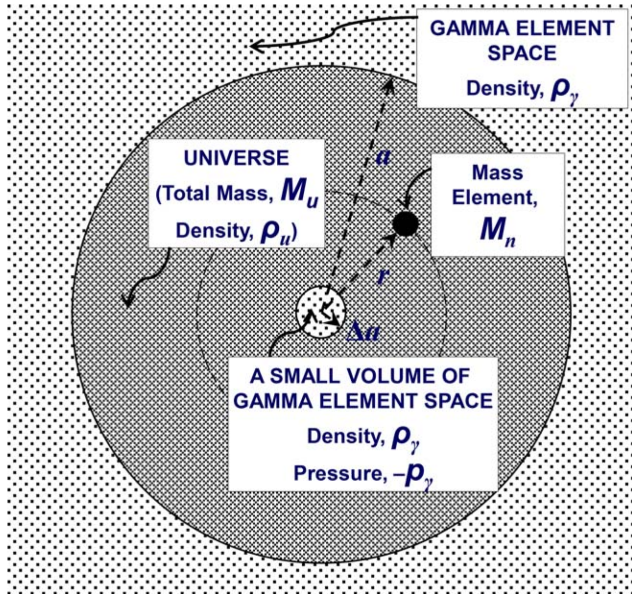

FIG. 1. (Color online) An elemental spacetime model for the universe (not to scale).

naturally assume that the γ-element space is perfect fluid with its behavior characterized by the mass density, ρ_γ, and isotropic pressure, p_γ (tensile pressure if $p_\gamma < 0$). To obtain the total pressure exerted by the gravity of the universe, we will consider, in particular, the pressure $p_{\gamma n}$ exerted by the mass M_n at a distance, r_n,

$$p_{\gamma n} = -\frac{G\rho_\gamma M_n}{r_n}\gamma_n, \tag{14}$$

where $\gamma_n = [1 - (\dot{r}_n/c)^2]^{-1/2}$. Now since p_γ is a scalar, we have the total pressure

$$p_\gamma = \sum_n p_{\gamma n}, \tag{15}$$

where the summation is for all the masses, M_n, in the universe (other than the small volume of Δa being considered). The summation may then be performed as integral

$$\begin{aligned} p_\gamma &= -G\rho_\gamma \int_0^a \rho_u \frac{4\pi r^2}{r}\frac{1}{\sqrt{1 - (\dot{r}/c)^2}}dr, \\ &= -4\pi G\rho_\gamma\rho_u\frac{c^2}{H^2}\left(1 - \sqrt{1 - \frac{H^2}{c^2}a^2}\right), \end{aligned} \tag{16}$$

where a denotes the radius of the universe measured from the center of the small volume of radius Δa being considered. The integration is performed over the universe, Δa to a, and we let $\Delta a \to 0$. We use the Hubble's empirical law $\dot{r} = Hr$ assuming H is constant throughout the universe at a particular epoch. The density of the universe, ρ_u (from the radius Δa to a,) is the sum of the density of the γ-elements, ρ_γ, which is constant and independent of a, and the density of all other matters, ρ_a, that is proportional to a^{-3}. Note the differences of notations between the present work and the ΛCDM model, $\rho_\gamma \equiv \rho_\Lambda/c^2 \equiv \rho_{\text{vac}}/c^2$, so $\Omega_\gamma \equiv \Omega_\Lambda$. ρ_m in the ΛCDM model may include observable as well as the nonobservable dark matters versus ρ_a in the present work may include only the observable matters. In the present work, ρ_{rad} (or Ω_{rad}) is ignored as it is small.

Equation (16) then presents a local sound wave speed,[16,17] or in this case what we might call the speed of the gravitational tensile pressure waves

$$c_g^2 = -\frac{\partial p_\gamma}{\partial \rho_\gamma} = 4\pi G\rho_u \frac{c^2}{H^2}\left(1 - \sqrt{1 - \frac{H^2}{c^2}a^2}\right) \text{ or if } c_g = c,$$
$$\frac{a^2}{c^2} = \frac{1}{H^2}\left[1 - \left(1 - \frac{H^2}{4\pi G\rho_u}\right)^2\right]. \tag{17}$$

If $c_g = c$, then Eqs. (16) and (17) yield

$$\rho_\gamma = -p_\gamma/c^2. \tag{18}$$

This is the barotropic equation of state. Wilczek[18] calls it a well-tempered equation.

If $\dot{a} = Ha \ll c$ or in the nonrelativistic limit, Eqs. (16) and (17) reduce, respectively, to

$$p_\gamma = -2\pi G\rho_\gamma \rho_u a^2 \tag{19}$$

and

$$c_g^2 = -\frac{\partial p_\gamma}{\partial \rho_\gamma} = 2\pi G\rho_u a^2 \text{ or if } c_g = c,$$
$$\frac{a^2}{c^2} = \frac{1}{2\pi G\rho_u}. \tag{20}$$

If $c_g = c$, the speed of light, then Eqs. (19) and (20) again yield Eq. (18), the well-tempered equation. The well-tempered equation is, therefore, Lorentz covariant.

The sound waves defined in Eqs. (17) and (20) are dilatational, a consequence of the γ-element space or the cosmological constant as discussed in Section II. To the author's knowledge, there is no analogous concept in GR, but if the cosmological constant is nonzero they must be present on top of the gravitational waves that are the perturbation radiation of the spacetime itself.[14,19]

VI. THE EXPANDING UNIVERSE ACCORDING TO THE SR-EXTENDED NEWTON'S LAW

We use Newton's shell theorem to calculate the total energy, the sum of the kinetic energy (K.E.) and the potential energy (P.E.), of the nth particle having mass, M_n, at the radius a from the center. Note that Newton's shell theorem ignores the gravitational pressure derived in Section V. (Such pressure is in equilibrium and does not contribute to the motion of the particle.) We may then write an equation for the total energy

$$\text{K.E.} + \text{P.E.} = \gamma M_n c^2 - M_n c^2 - \gamma G M_u M_n/a = \text{constant} \equiv -(\kappa/2)M_n c^2, \tag{21}$$

where

$$\gamma = \left[1 - (\dot{a}/c)^2\right]^{-1/2}, \tag{22}$$

and M_u is the total mass within the sphere of the radius $a(t)$, a function of time. We have given the constant an ansatz, $-(\kappa/2)M_n c^2$. In the Newtonian gravitation, if $\kappa < 0$ the kinetic energy provides more than the escape velocity; if $\kappa = 0$ the kinetic energy and the gravitational potential energy balances (flat); and if $\kappa > 0$ the kinetic energy is insufficient for M_n to escape from the gravity of M_u. Equations (21) and (22) yield

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{(1-\kappa/2)^2}\left[\frac{2GM_u}{a^3} - \frac{1}{c^2}\frac{G^2M_u^2}{a^4} - \frac{\kappa c^2(1-\kappa/4)}{a^2}\right]. \tag{23}$$

For sufficiently small $|\kappa|$ or $|\kappa| \ll 1$, the above approximates to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2GM_u}{a^3} - \frac{G^2M_u^2}{c^2a^4} - \frac{\kappa c^2}{a^2}. \tag{24}$$

We note

$$M_u = M_\gamma + M_a, \tag{25}$$

where M_a is the mass of all matters other than M_γ, the mass of the γ-elements, within the sphere of the radius a. According to the model shown in Fig. 1, we have the following relationships:

$$\rho_\gamma = M_\gamma \Big/ \left(\frac{4\pi}{3}a_3\right) = \text{constant}, \tag{26}$$

$$M_a = \frac{4\pi}{3}a_3\rho_a = \text{constant}, \tag{27}$$

$$\rho_u = M_u/\left(\frac{4\pi}{3}a^3\right) = \rho_\gamma + \rho_a. \tag{28}$$

We then get from Eq. (24),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_u}{3} - \frac{16\pi^2G^2\rho_u^2}{9}\frac{a^2}{c^2} - \frac{\kappa c^2}{a^2}, \tag{29}$$

which is the SR-extended FRW equation (hereinafter denoted as eFRW) according to the SR-extended Newtonian gravity, i.e., eFRW-Newtonian or eF$_\text{N}$. Equation (29) reduces to the usual FRW equation if the second term is ignored. This extra term is present because we used the full SR kinetic energy term $\gamma M_n c^2 - M_n c^2$ and SR-extended Newton's law, Eq. (12), instead of their slow-motion approximations, $(1/2)M\dot{a}^2$ and the normal Newton's law, respectively.

Equation (29), together with the relativistic gravitational sound wave, Eq. (17), and by setting $c_g = c$ and $\kappa = 0$, are solvable for ρ_u and a, with the results,

$$\rho_u(\text{eF}_\text{N}) = \frac{H^2}{2\pi G}; \quad a(\text{eF}_\text{N}) = \frac{\sqrt{3}}{2}\frac{c}{H}. \tag{30}$$

For $H = H_0$, Eq. (30) gives $\rho_{\rm crit} = H_0^2/(2\pi G)$ versus Equation (3).

If the second (order) term is ignored, Eq. (29) reduces to the (nonrelativistic) FRW equation, Eq. (1). This together with the nonrelativistic gravitational sound wave, Eq. (20), and with $c_g = c$ and $\kappa = 0$, gives

$$\rho_u(\mathrm{F}) = \frac{3H^2}{8\pi G}; \quad a(\mathrm{F}) = \frac{2}{\sqrt{3}}\frac{c}{H}, \tag{31}$$

where (F) denotes the values according to the usual FRW equation. For $H = H_0$, Eq. (31) gives $\rho_{\rm crit}$ of Eq. (3). Note that neither Eq. (30) nor Eq. (31) demands the knowledge of the composition of ρ_u or the velocity of the expansion. The difference is only caused by the kinematics of the SR, specifically the second order term of Eq. (29).

VII. THE EXPANDING UNIVERSE ACCORDING TO THE GENERAL RELATIVITY WITH SPECIAL-RELATIVISTIC DYNAMIC EFFECTS

The Robertson–Walker metric[2–4] may be written in the Cartesian coordinates

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + a^2(t)\left(dx_i^2 + \kappa\frac{x_i^2 dx_i^2}{1-\kappa r_2}\right), \tag{32}$$

with $\mu,\nu = 0,1,2,3, \;\; i,j = 1,2,3, \;\; r^2 \equiv x_i^2 = x^2 + y^2 + z^2$, and the nonzero components of the metric tensor, $g_{\mu\nu}$,

$$g_{00} = -1; \quad g_{ii} = a^2(t)\left(1 + \kappa\frac{x_i^2}{1-\kappa r_2}\right)\,(\text{no sum}). \tag{33}$$

The nonzero components of Ricci tensor, $R_{\mu\nu}$, are[14]

$$\begin{aligned} R_{00} &= -3\ddot{a}/a;\\ R_{ij} &= (1/a^2)(\ddot{a}a + 2\dot{a}^2 + 2\kappa)g_{ij}; \end{aligned} \tag{34}$$

and

$$R = (6/a^2)(\ddot{a}a + 2\dot{a}^2 + 2\kappa). \tag{35}$$

The Einstein equation may be written in the form

$$R_{\mu\nu} = \frac{8\pi G}{c^2}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right), \tag{36}$$

where $T_{\mu\nu}$ is the stress-energy tensor with T its trace.

The stress energy tensor for the perfect fluid may be written

$$T_{\mu\nu} = (\rho_u + p/c^2)U_\mu U_\nu + p g_{\mu\nu}, \tag{37}$$

where U_μ is the mean four velocity of the galaxies near the event of interest (Ref. 19, p. 713). Our interest is to describe the whole flat universe by a single expansion factor or the radius. With $\beta = \dot{a}/c$ and $\gamma = (1-\beta^2)^{-1/2}$, we have the four-velocity in the comoving frame

$$U^\mu = \frac{\partial x^\mu}{\partial\tau} = \frac{\partial x^\mu}{\partial t}\frac{\partial t}{\partial\tau} = \gamma u^\mu = \gamma(c,0,0,0). \tag{38}$$

The Lorentz factor, γ, is essential since without it, the velocity itself is not in general a four vector. It brings the dynamic effect for the expanding universe. If we further assume $\gamma = 1$, we will eventually arrive at the FRW equation which is nonrelativistic. In this sense, the term special-relativistic also means dynamic, and the two terms may be interchangeable. We then have from Eqs. (37) and (38),

$$\begin{aligned} T_{00} &= (\rho_u c^2 + p)\gamma^2 - p = \rho_u c^2\gamma^2 + p(\gamma^2 - 1)\\ &= \gamma^2(\rho_u c^2 + \beta^2 p); \quad T_{ij} = p g_{ij};\\ T &= T^\mu_\mu = T_{\mu\nu}g^{\mu\nu} = T_{00}g^{00} + T_{ij}g^{ij}\\ &= -\gamma^2(\rho_u c^2 + \beta^2 p) + 3p. \end{aligned} \tag{39}$$

One can then find the 00 equation

$$-3\frac{\ddot{a}}{a}\gamma^2 = 4\pi G\gamma^2\left[\rho_u + (3 - 2\beta^2)\frac{p}{c^2}\right], \tag{40}$$

and the ij equation

$$\frac{\ddot{a}a + 2\dot{a}^2 + 2\kappa}{a^2}\gamma^2 g_{ij} = 4\pi G\gamma^2\left[\rho_u - (1 - 2\beta^2)\frac{p}{c^2}\right]g_{ij}, \tag{41}$$

where we can drop g_{ij}. The dimensions of the term $\ddot{a}/a, (\dot{a}/a)^2$, and a^{-2}, [1/time]^2 or [1/length]^2, mean we must add γ^2 to the left hand side to ensure the Lorentz covariance.[6] From Eqs. (40) and (41),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_u + \beta^2\frac{p}{c^2}\right) - \frac{\kappa c^2}{a^2}. \tag{42}$$

Note that if $\beta = 0$, Eq. (42) is the FRW equation. We are now ready to solve for ρ_u and a from Eqs. (42) and (17). We must, however, first recognize that

$$p_\gamma = 3p. \tag{43}$$

This is because p_γ is the sum of pressures in all radial directions in polar coordinates, which is the same as the sum of the pressures, p, in all three rectangular coordinate directions. By assuming $C = \kappa = 0$, we finally get

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_u\left(1 - \beta^2\frac{\Omega_\gamma}{3}\right). \tag{44}$$

This is the eFRW equation according to the GR, i.e., eFRW-GR or eF$_{\rm G}$ or eF$_{\rm G}(\beta,\Omega_\gamma)$, which compares with Eq. (29). If $\beta = 0$ or $\Omega_\gamma = 0$, Eq. (44) again reduces to the (nonrelativistic) FRW equation. By the use of $\rho_\gamma = \rho_u - \rho_a$, we then get

$$\begin{aligned} \rho_u(\beta,\Omega_\gamma) &= \frac{3H^2}{8\pi G}\left(1 - \beta^2\frac{\Omega_\gamma}{3}\right)^{-1} \quad \text{or}\\ \rho_u(\beta,\rho_a) &= \left(\frac{3H^2}{8\pi G} - \frac{\beta^2}{3}\rho_a\right)\left(1 - \frac{\beta^2}{3}\right)^{-1}. \end{aligned} \tag{45}$$

Equation (17) gives the corresponding solutions for the radius of the universe. With the notations $\rho_u(\mathrm{eF_G}) \equiv \rho_u[\mathrm{eF_G}(\beta,\Omega_\gamma)] \equiv \rho_u(\mathrm{eF_G},\beta,\Omega_\gamma)$ and $a(\mathrm{eF_G}) \equiv a[\mathrm{eF_G}(\beta,\Omega_\gamma)] \equiv a(\mathrm{eF_G},\beta,\Omega_\gamma)$, we get various limit solutions for Eq. (45)

$$\rho_u(\mathrm{eF_G},1,1) = \frac{9}{16}\frac{H^2}{\pi G}; \quad a(\mathrm{eF_G},1,1) = \frac{2\sqrt{14}}{9}\frac{c}{H}, \tag{46}$$

and

$$\rho_u(\mathrm{eF_G},0,\Omega_\gamma) = \frac{3}{8}\frac{H^2}{\pi G}; \quad a(\mathrm{eF_G},0,\Omega_\gamma) = \frac{2\sqrt{2}}{3}\frac{c}{H}. \tag{47}$$

The second of Eq. (45) may be used to calculate $\rho_u(\beta)$, a function of the velocity, when H and ρ_a are known by measurements.

VIII. THE DCD AND ORIGIN OF THE DARK MATTER

Equation (45) defines the DCD for the flat universe at the present epoch

$$\rho_{crit} = \rho_u \text{ for } \kappa = 0 \text{ and } H = H_0. \tag{48}$$

The SCD, Eq. (3), is a special case denoted in this section as $\bar{\rho}_{crit}$. Below for brevity, we drop the subscript 0 and interchangeably use ρ_u and $\rho_{\rm crit}$ unless otherwise specified. The limit values of the critical density and the corresponding radius of the universe may then be found for various analytical cases from Eqs. (30), (31), (46), (47), (17), and (20) as listed in Table I.

There is a trivial GR solution of Eq. (45) which shows that when $\beta = 0$ any combinations of Ω_γ and Ω_a is valid as long as $\Omega_\gamma + \Omega_a = 1$. For more general cases, $0 < \beta \le 1$ and $0 < \Omega_\gamma \le 1$, Eq. (45) gives the values of ρ_u and a as functions of β and Ω_γ or of β and ρ_a. Given ρ_a, we can calculate ρ_u, then $\rho_\gamma = \rho_u - \rho_a$. By $\Omega_\gamma = \rho_\gamma/\rho_u$, $\Omega_a = \rho_a/\rho_u$, it then guarantees $\Omega_u = \Omega_a + \Omega_\gamma = 1$ as it should. In particular, if the density of the observable baryonic matter, ρ_b, is fixed according to the Wilkinson microwave anisotropy probe (WMAP) measurement,

$$\rho_a \approx \rho_b \approx 0.418 \times 10^{-27}\ \mathrm{kg/m^3}; \tag{49}$$

under this constraint the admissible pairs of β and Ω_γ may then be found numerically. The numerical solution for Eq. (45) for the (β,Ω_γ) pair, (0.864, 0.965), is of particular interest and also listed in Table I. This solution will be discussed below and in Section IX.

TABLE I. Critical density and radius for various analytical cases. F stands for FRW equation; $\mathrm{eF_N}$ for eFRW equation within the SR-extended Newtonian gravity; and $\mathrm{eF_G}(\beta,\Omega_\gamma)$ for eFRW equation within the GR.

Analytical case	$\rho_{\rm crit}/[H^2/(\pi G)]$	$a_{\rm crit}/(c/H)$
F	3/8	$2/\sqrt{3}$
$\mathrm{eF_N}$	1/2	$\sqrt{3}/2$
$\mathrm{eF_G}$ $(0,\Omega_\gamma)$	3/8	$2\sqrt{2}/3$
$\mathrm{eF_G}$ (0.864, 0.965)	≈0.494	≈0.870
$\mathrm{eF_G}$ $(\sqrt{3}/2, 1)$	1/2	$\sqrt{3}/2$
$\mathrm{eF_G}$ (1, 1)	9/16	$2\sqrt{14}/9$

From Eq. (45), one may also write

$$\rho_u = \left(1 - \frac{\beta^2}{3}\bar{\Omega}_a\right)\left(1 - \frac{\beta^2}{3}\right)^{-1}\bar{\rho}_u; \text{ where} \quad \bar{\Omega}_a = \rho_a/\bar{\rho}_u; \quad \bar{\rho}_u = \frac{3H^2}{8\pi G} \tag{50}$$

which gives the relationship between the DCD, $\rho_{\rm crit} = \rho_u$, and the SCD, $\bar{\rho}_{\rm crit} = \bar{\rho}_u \equiv (3H^2)/8\pi G$. Suppose we analyze cosmic data without the benefit of Eq. (45) but only in the FRW framework or ΛCDM. One will then use the SCD for normalization, hence from Eq. (45),

$$\bar{\Omega}_\gamma = \frac{\beta^2/3}{1-\beta^2\bar{\Omega}_a/3}\bar{\Omega}_a^2 - \frac{1}{1-\beta^2\bar{\Omega}_a/3}\bar{\Omega}_a + \frac{1-\beta^2/3}{1-\beta^2\bar{\Omega}_a/3}, \tag{51}$$

where $\bar{\Omega}_\gamma = \rho_\gamma/\bar{\rho}_{\rm crit}$; $\bar{\Omega}_a = \rho_a/\bar{\rho}_{\rm crit}$. This compares with the WMAP empirical relation, a data fit that follows a slightly curved line:[20]

$$\bar{\Omega}_\gamma = 0.0620\bar{\Omega}_a^2 - 0.825\bar{\Omega}_a + 0.947. \tag{52}$$

Both Eqs. (51) and (52) generally give $\bar{\Omega}_a + \bar{\Omega}_\gamma \neq 1$.

In Fig. 2, $\Omega_\gamma(\bar{\Omega}_\gamma)$ is shown as a function of $\Omega_a(\bar{\Omega}_a)$ (WMAP notations are Ω_Λ and Ω_m, respectively; the bar has

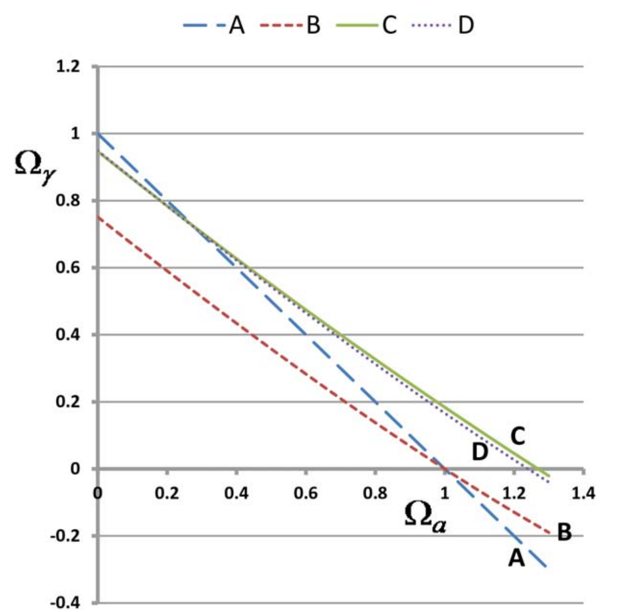

FIG. 2. (Color online) $\Omega_\gamma(\bar{\Omega}_\gamma)$ is shown as a function of $\Omega_a(\bar{\Omega}_a)$ (WMAP notations are Ω_Λ and Ω_m, respectively; the bar has been dropped in the axis titles) to reveal the characteristics of the ΛCDM model in view of present model. Line A shows the trajectory, $\Omega_\gamma = 1 - \Omega_a$, where $\Omega_\gamma = \rho_\gamma/\rho_u$; $\Omega_a = \rho_a/\rho_u$ are the solutions of $\rho_u(\beta,\Omega_\gamma) = 3H^2/(8\pi G)(1-\beta^2\Omega_\gamma/3)^{-1}$ with $\beta = 0.864$ fixed and $\rho_u(\beta,\Omega_\gamma) \equiv \rho_{\rm crit}$ is the DCD. Curve B is the same except the solutions are normalized by the SCD, $\bar{\Omega}_\gamma = \rho_\gamma/\bar{\rho}_u$; $\bar{\Omega}_a = \rho_a/\bar{\rho}_u$; $\bar{\rho}_u \equiv \bar{\rho}_{\rm crit} = 3H^2/(8\pi G)$, which then result in a near-quadratic relationship, $\bar{\Omega}_\gamma = [(\beta^2/3)/(1-\beta^2\bar{\Omega}_a/3)]\bar{\Omega}_a^2 - [1/(1-\beta^2\bar{\Omega}_a/3)]\bar{\Omega}_a + (1-\beta^2/3)/(1-\beta^2\bar{\Omega}_a/3)$. Curve C is the quadratic relationship empirically obtained by WMAP, $\bar{\Omega}_\gamma = 0.0620\bar{\Omega}_a^2 - 0.825\bar{\Omega}_a + 0.947$. Curve D is the same as Curve B except translated horizontally by a constant, $\Delta\Omega_a = \Omega_c = 0.240$, i.e., by the amount of the CDM, to overlap Curve C almost precisely (within 1.8% near $\Omega_a = 1$).

been dropped in the axis titles) to reveal the characteristics of the ΛCDM model in view of present model. Line A shows the trajectory, $\Omega_\gamma = 1 - \Omega_a$, where $\Omega_\gamma = \rho_\gamma/\rho_u$; $\Omega_a = \rho_a/\rho_u$ are the solutions of $\rho_u(\beta, \Omega_\gamma) = 3H^2/(8\pi G)(1 - \beta^2\Omega_\gamma/3)^{-1}$, Eq. (45), with $\beta = 0.864$ fixed and $\rho_u \equiv \rho_{\text{crit}}$ is the DCD. Curve B is the same except the solutions are normalized by the SCD, $\bar{\Omega}_\gamma = \rho_\gamma/\bar{\rho}_u$; $\bar{\Omega}_a = \rho_a/\bar{\rho}_u$; $\bar{\rho}_u \equiv \bar{\rho}_{\text{crit}} = 3H^2/(8\pi G)$, which then result in a near-quadratic relationship, Eq. (51). Curve C is the quadratic relationship empirically obtained by WMAP, Eq. (52). If Curve B is allowed to translate horizontally by a constant, $\Delta\Omega_a = 0.240$, it then becomes Curve D, which is found to overlap Curve C almost precisely (within 1.8% near $\Omega_a = 1$.) This translation is accomplished by replacing Ω_a in Eq. (51) with $\Omega_a - 0.240$. Clearly, this translation represents the CDM, $\Delta\Omega_a = \Omega_c = 0.240$, but is it real? The answer is no, because Curve B is not real, only Line A is the real solution to Eq. (45). So how do we arrive at Line A from the empirical curve C? First take the CDM, $\Omega_c = 0.240$, out of Curve C to bring it back to match with Curve B, replace the SCD used originally with the DCD, we will then arrive at the data fit curve that matches the predicted Line A. This reverse process shows that the hypothetical CDM is only an artifact of the FRW equation which is insufficient for the description of the dynamic effects. We have finally identified, almost certainly according to the present analysis, the source of the CDM at least on a large scale.

According to the Hubble's law, $H = \dot{a}/a$, the velocity of the expansion would begin to exceed the speed of light, $\dot{a} = c$, at the critical radius, $a_{\text{crit}} = c/H$. According to the FRW expansion, Table I or Eq. (31), $a(\text{F}) = (2/\sqrt{3})a_{\text{crit}}$, the edge of the universe is currently expanding at the speed greater than that of the light. All other cases summarized in Table I predict $a < a_{\text{crit}}$. When dynamic effects are included, the edge of the universe is predicted to expand at the speed less than that of light, as it should be according to the SR.

IX. DARK ENERGY AND DARK MATTER—NUMERICAL EXAMPLES

A. The universe

Table II shows an inventory of the cosmic mass densities: the observable matter, dark energy, and dark matter. Those reported by WMAP[20–22] and Planck projects[23,24] both use the ΛCDM model and are close to each other. We only use the former in this table. All calculations used the observable matter density, Eq. (49), and the Hubble constant,

$$H_0 \approx 2.25 \times 10^{-18}\,\text{s}^{-1}\ (\approx 69.3\,\text{km/s/Mpc}), \tag{53}$$

both from the WMAP. The observable matter includes the baryonic matter; other small quantities such as the cosmic microwave background (CMB) radiation[25] have been ignored.

The second column shows $\rho_u(\text{F})$ and its breakdown according to the WMAP +eCMB+BAO+H_0 including dark matter. The third and fourth columns show the breakdown by the eFRW-Newtonian (eF_N) and by eFRW-GR (eF_G), respectively. $\rho_u[\text{eF}_\text{G}(0.864, 0.965] \approx 0.494H^2/(\pi G)$ matches $\rho_u(\text{eF}_\text{N}) = (1/2)H^2/(\pi G)$ within ≈1.2%. The dynamic models, both $\rho_u(\text{eF}_\text{N})$ and $\rho_u[\text{eF}_\text{G}(\beta, \Omega_\gamma)]$, are able to fit the cosmological data with two parameters, $\Omega_u = \Omega_b + \Omega_\gamma = 1$, rather than three, $\Omega_u = \Omega_b + \Omega_c + \Omega_\gamma = 1$, i.e., without the CDM. The last column is likely the most accurate prediction for the breakdown of the density of the universe.

TABLE II. Cosmic mass density inventory. The second column shows the breakdown by WMAP +eCMB+BAO+H_0 (WMAP.) The third and the fourth are the breakdowns by the eFRW-Newtonian (eF_N) and by the eFRW-GR (eF_G), respectively. Note that the latter two do not require the CDM to fit the cosmic data and agree with each other within 1.4%.[a]

	F ΛCDM (WMAP)	eF_N	eF_G (0.864, 0.965)
$\beta = \dot{a}/c$	~0	N/A	0.864
ρ_b (Baryon)	0.418	0.418	0.418
Ω_b	0.0463	0.0347	0.0352
ρ_γ (Dark Energy)	6.44	11.62	11.46
Ω_γ	0.714	0.965	0.965
ρ_c (CDM)	2.17	0	0
Ω_c	0.240	0	0
ρ_u (Total)	9.03	12.04	11.88
$\rho_u/[H^2/(\pi G)]$	3/8	1/2	0.494

[a] $H = 69.3$ km/s/Mpc (WMAP); Unit of ρ is $10^{-27}\,\text{kg/m}^3$.

B. Rotation curves

The need for the dark matter originates from the observations of the spiral galaxies or the rotation curves of them[26–28] Most cosmological observations are satisfactorily described by the ΛCDM model with $\Omega_b \sim 0.05$, $\Omega_c \sim 0.25$, and $\Omega_\gamma(\equiv \Omega_\Lambda) \sim 0.7$. This includes[28] the study of the supernovae, the spiral galaxies, galaxy clusters, gravitational lens, CMB, and large scale structures such as Lyman-alpha forest.[29]

Recently, however, McGaugh *et al.*[30] followed 153 various galaxies and reported a definitive correlation between the radial acceleration traced by rotation curves and that predicted by the observed distribution of baryons. The dark matter contribution is then fully specified by that of the baryons, in support of the present theory. According to the present calculations, the same effect of the special relativity in the form of the DCD must be present uniformly in the entire universe. Any local behavior such as that of the spiral galaxies must also take it into account even though the rotation curves show the local velocities much less than the speed of light. This is born out in Eq. (38) which states the four-velocity remains the same for an observer anywhere in the universe. The DCD may help explain the seemingly anomalous rotation curves of the galaxies or the gravitational lensing. This warrants a detailed study of the local behaviors which, however, is beyond the scope of this paper.

X. CONCLUSION

An analysis according to the principles of special and general relativity and less restrictive Newtonian gravity proves the dynamic effects to be substantial for the expanding universe. With the resulting DCD, typically greater than the SCD, I am able to identify the hypothetical CDM as being an artifact of the FRW equation that is insufficient to describe the dynamic effects. With the included special-relativistic dynamic effects, I can now predict the cosmic data with two parameters, matter and the cosmological constant, without the CDM at least on a large scale.

The SR extension of the Newton's gravity, Eq. (29), is an asymptotic approximation of the GR. The second method is the SR extension of the FRW equation derived within the framework of GR, Eq. (44). The dynamic critical densities they predict for our expanding universe agree within 1.4% of each other, a convincing result.

Other critical elements of this work include the gravitational sound wave speeds, both SR-extended and nonrelativistic, Eqs. (17) and (20), respectively, and the barotropic equation of state, Eq. (18). The derivation of these equations assumes the presence of mass anywhere in the universe, a trait of the cosmological constant, dark energy, or the γ-elements, all referring the same thing in various units.

I should mention some of the work that also attempt to extend the Newtonian gravity to include modified or relativistic features, particularly the modification of the Newtonian dynamics (MOND)[31–34] and a special-relativistic correction of Newton's law.[35,36] Unfortunately, they tend to be either a phenomenological formulation or outside the Newtonian or special and general relativistic principles.

To the author's knowledge, this is the first time (a) the effect of the special-relativistic dynamics has been accounted for in the FRW equation both by the SR-extended Newton's law and within the framework of the GR and (b) a negative pressure of the space is calculated by the use of both the Newton's law and the SR-extended Newton's law.

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